Cost Pass through: An Impediment to Innovation?

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Abstract

This paper investigates a model of endogenous innovation which results in product differentiation. Prior to innovation the firms supplied an entirely homogeneous product. Unlike other models of product differentiation, the cost of innovation is passed on to consumers in the form of increased prices. However, some consumers do not immediately recognize the innovation and/or its benefits. Such consumers express an unwillingness to pay for this innovation by purchasing from a firm who does not innovate. A portion of this same fraction of consumers eventually realize the innovation and resume purchases with the innovative firm with an acceptance of the higher price. Under reasonably broad conditions it is shown that both the innovator's and the non-innovator's profits will increase after innovation. Despite the friction created by switching consumers, continued innovation on the part of the first firm to innovate is optimal.
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1 Introduction

The notion that increased competition lowers profitability is widely supported by empirical evidence and has been treated as a stylized fact of economics for many years (see Tirole, 1988). Since competition in an oligopoly market can be fierce, strategic quality differentiation is often used to mitigate the effects of competition on profitability. However, the costs inherent in this kind of innovation in terms of quality[1] - to differentiate one firm's product from it's competitors' - often results in increased prices that consumers might be unwilling to accept. This paper analyzes the effect of such innovation on firms' profitability and the degree of competition.

Bodnar, Dumas, and Marston (2002) noted that pricing directly affects profitability and the extent of the cost pass through[2] will affect the profitability of a firm. According to Nas and Leppalahti (1997), firms can raise prices on the basis of their performance, improvements in their product and its 'imperfect competition' advantage, which will lead to improved returns on sales or assets. Geroski (1993, 1995) proposed that the performance of innovative firms is more persistent, and that they are less vulnerable, for example, to cyclical downturns. Therefore, innovators who can successfully pass on costs to customers are likely to be more profitable, and have more flexibility when adjusting to external incidents like demand and supply shocks.

Despite these advantages, firms must carefully consider the cost of innovation. Nas and Leppalahti (1997) highlighted that using innovation to differentiate the company's products/services from their competitors' requires the creation of tangible and intangible assets which increase production costs. From this perspective, innovating firms will not necessarily be more profitable, but they will be more likely to survive and grow. Several studies have focused on cost pass-through rates (CPTR) under varying market conditions. According to Cotterill and Kim (2006), the CPTR is greater when demand is more inelastic and supply more elastic. Bulow and Pfleiderer (1983) and Stiglitz (1988) show that a monopolist facing a linear demand curve with constant marginal costs will pass through 50 percent of cost changes, while under perfect competition, the CPTR would be determined by the relative demand and supply elasticities.

A key determinant of innovation is the willingness of consumers to pay a higher price for the improved products. Narajabad and Watson (2008) found that the key influences on an industry's innovation rate is determined by the heterogeneity in consumer tastes and the cost of switching. They also demonstrate that changes in quantity can have different effects on the innovation rate. A

[1]Note that in a world of horizontal differentiation firms may not need to incur costs to differentiate products. As such, we consider our model one of vertical differentiation.

[2]This measures the portion of a change in input cost that is transferred to the price faced by consumers.

reduction in switching cost, for example, may either raise or lower innovation depending on the current degree of taste heterogeneity. In explaining the rate of adoption of an innovation Rogers (1995) also noted that buyers' reaction to the innovation (idea or object) will be determined by how individuals or agencies perceive the innovated product.

In an attempt to overcome the impediments to innovation, Penrose (1959) suggests that organizations embark on new products. Additionally, firms may develop new uses for old products so that they can produce goods or services with uses which consumers are unaware but will find useful and be willing to pay. Piana (2004) concluded that sales are affected in three ways after innovation. Some customers will not buy the product because it does not satisfy their minimum requirement. Other customers may decide to stay with their current provider even though the overall quality improvement of another provider is good. This is so as it is not worth switching to another provider. Furthermore, a third set of customers will recognize that the product offers 'value-for-money' and switch from their provider. This discussion feeds perfectly into one of our main premises. That is, the idea that firms embark on the innovation process knowing that they will face some active consumer resistance. However, there is also the anticipation that some fraction of those resistant consumers will eventually accept the product[3].

One of the first attempts to model innovation which results in product differentiation, an important determinant of profitability, was developed by Hotelling in 1929. Hotelling's model was premised on horizontal product differentiation and showed that if firms are given the choice, they would choose to minimally differentiate themselves. Hotelling's (1929) seminal work on spatial competition resulted in a rich and diverse literature on the theory of location[4]. Although these theories provided important insights into imperfect competition, the impact of price competition on product selection was ignored, limiting the applicability of the model (Chaplin and Nalebuff, 1986).

The premise of minimal product differentiation was questioned by D'Aspremont, Gabszewicz and Thisse (1979) under a quadratic cost structure where it was shown that firms can achieve maximal product differentiation. Thomas (2011) notes that D'Aspremont, Gabszewick and Thisse (1979) and Salop (1979) led the way for the development of models of vertical product differentiation. However, Bresnahan's discrete choice model of vertical product differentiation has been one of the most popular adapted methodologies in the analysis of vertical product differentiation. It must be noted that the Bresnahan model has its

[3]Consumer reluctance to accepting innovative products can be vindicated by some real world examples. These are cases where the initial product resulting from the innovation process does not function optimally. Two recent episodes are Apples I-Phone G; and Microsoft's Windows Vista.

[4]Again, Hotelling's work is about horizontal differentiation and therefore may not be applicable to the discourse of this paper

roots in the work of Shaked and Sutton (1982) and (1983), among other pieces of work which led the revolution in the vertical differentiation literature.

This study takes the approach of the model presented by Bresnahan (1987), being a model which succinctly captures most of the germane characteristics of models of vertical differentiation. Bresnahan (1987) deploys a discrete choice model of vertical product differentiation to study the automobile industry in America. In this model consumers care about the quality of the product and as a result consumers' utility is modeled depending on the type of good.

This paper is premised on the assumption that before innovation, firms produce an entirely homogenous good at the same marginal cost. Thus the results of the Bertrand paradox, which states that firms set price equal to marginal cost and make zero profits holds. After innovation, the innovator's marginal cost is increasing with innovation, while the non-innovator's marginal cost remains the same. The remainder of the paper is organized as follows: Section 2 presents The Model; Section 3 details some Comparative Statics analysis; and Section 4 concludes.

2 The Model

Prescott and Visscher (1977), developed a model of vertical product differentiation, where consumers are uniformly distributed along a unit interval, in that, consumers differ in their marginal utility of an increase in quality. In addition, the product choice is discrete, and firms' physically homogeneous products are viewed as vertically differentiated in accordance with the consumer preference — the location on the unit interval. This model was later used by Bresnahan (1987) and will be the one explored here. The analysis employs a two stage game where one firm chooses to innovate and the other firm chooses not to in the first stage. In the second stage firms compete in prices. This means that types (innovated product and non innovated product) are exogenous variables that affect prices in the second stage.

We posit that marginal costs are not only different but the innovators' marginal cost is increasing with innovation while, the non-innovators' marginal cost remains the same after innovation.

To simplify the analysis, the number of firms is restricted to two and the firm that innovates is classified as the high type firm denoted by H; while the other firm is known as the low type firm denoted by L. Now, to formalize the model, consider a two stage game where firms, $i \epsilon I = \{1, 2\}$, make the following choices:

Stage 1:

One firm chooses to innovate (high type firm) and the other does not (low type firm).

Stage 2:
Both firms choose the price at which to sell output.

The marginal cost faced by the innovator is a strictly increasing function of innovation, thus it's price will be greater than that of the non-innovator's, that is $P_H > P_L$. This shows that the innovated product is superior to the non innovated product. We also assume that consumers are atomistic. They make their decision given the type and prices of both goods. The model assumes that the utility gained from consuming an outside good is equal to zero. In addition, a consumer generates his/her payoff from consuming a good depending on his/her preferences. Thus a consumer of type $\widehat{V}$, has payoff function:

$$\begin{cases} \widehat{V}H - P_H & \text{if } H \text{ is chosen} \\ \widehat{V}L - P_L & \text{if } L \text{ is chosen} \end{cases}$$

$\widehat{V}$ is the preference parameter that is uniformly distributed on the unit interval. That is, $\widehat{V} \sim v[0,1]$. First, we solve for the consumer who is indifferent between the two firms.

$$\widehat{V}H - P_H = \widehat{V}L - P_L$$
$$\widehat{V}H - \widehat{V}L = P_H - P_L$$
$$\widehat{V}(H - L) = P_H - P_L$$

$$\Leftrightarrow V = \frac{P_H - P_L}{(H - L)} \geq 0. \tag{1}$$

The solution for the indifferent consumer is used to derive the demand functions, which will be denoted as $D_i(p)$ for firm i, where i= (H,L).

$$D_H(P_H, P_L) = \begin{cases} 0 & \text{if } V \geq 1 \\ 1 - V & \text{if } 0 < V < 1 \\ 1 & \text{if } V \leq 0 \end{cases}$$

$$D_L(P_H, P_L) = \begin{cases} 1 & \text{if } V \geq 1 \\ V & \text{if } 0 < V < 1 \\ 0 & \text{if } V \leq 0 \end{cases}$$

Our analysis employs the case where $0 < V < 1$. Thus, firms have stage 2 profits stated as follows:

Profits Before Innovation

$$\prod_H(P_H, P_L) = (P_H - C_H)[D_H(P_H, P_L)]$$

$$\prod_L(P_H, P_L) = (P_H - C_L)[D_L(P_H, P_L)]$$

Prior to innovation we assume that marginal costs for both firms is the same but would vary with quality improvements[5]. As such, profits prior to innovation are characterized by the Betrand Paradox, which states that both firms set price equal to marginal cost ($P = MC$) and make zero profits. That is $\pi_H = \pi_L = 0$, $P_H = P_L$ and $C_H = C_L$.

Profits After Innovation

The literature on maximal product differentiation suggests that some customers will continue purchasing the increased quality product even if there is a price increase. However, in reality customers are not naive and thus will react to an increase in price. Rogers (1995) suggests that whether or not consumers switch depends on how they perceive the innovated product. Some customers will remain with the innovator as they will immediately recognize the value of the innovated product. In this paper Z captures the fraction of consumers who immediately recognize the value of the innovation and stay with the innovator.

The remaining consumers will switch to the non-innovator for a number of reasons; some believe that there is no difference between the new and the original product and thus feel as if the innovator is trying to cheat them. Some consumers will switch because the new product is too expensive and the non innovated product satisfies their minimum requirement. $(1 - Z)$ therefore represents the consumers who switch to the non-innovator after innovation. Some of these consumers will eventually recognize the value of innovation and thus return to the innovator. In this context let λ be the mean rate of arrival of consumers who return to the innovator, where $0 \leq \lambda \leq 1$. Therefore, without loss of

[5] These costs vary because of increasing input cost and technological changes.

generality let H denote the innovator firm and let L denote the non-innovator. Consequently, profits are given as follows:

$$\Pi_H(P_H, P_L) = Z(P_H - C_H)D_H + (1-Z)\lambda D_H(P_H - C_H) \tag{2}$$

$$\Pi_L(P_H, P_L) = (1-Z)(1-\lambda)(P_L - C_L)D_H + (P_L - C_L)[D_L(P_H, P_L)] \tag{3}$$

Where C_i is the marginal cost faced by each firm. As a result of the increasing cost with innovation, the innovator has to pass on some or all of the cost to it's consumers and thus charges a higher price.

Furthermore, under the assumption that price is strictly greater than marginal cost, the first order conditions provide the solution for the Nash equilibrium in prices stated as follows:

$$P_H = \frac{(H-L) + 2C_H + C_L}{3} + \frac{(H-L)}{3(1-B)} \tag{4}$$

where $B \equiv (1-Z)(1-\lambda)$

and

$$P_L = \frac{H - L + C_H + 2C_L}{3} - \frac{B(H-L)}{3(1-B)} \tag{5}$$

Proof: The first order conditions are[6]:

$\Pi_H = A(P_H - C_H)\left[\frac{(H-L)-(P_H-P_L)}{H-L}\right]$ Where $A \equiv Z + (1-Z)\lambda$

$$\frac{\partial \Pi_H}{\partial P_H} = A\left[\frac{(H-L)-(P_H-P_L)}{H-L} + P_H\left(\frac{-1}{H-L}\right) - C_H\left(\frac{-1}{H-L}\right)\right] = 0$$

[6]Detailed proof in Appendix A

$\Longleftrightarrow$

$$P_H = \frac{(H-L)+P_L+C_H}{2} = \frac{H-L+C_H}{2} + \frac{P_L}{2} \tag{6}$$

$$\Pi_L = (1-Z)[1-\lambda]D_H(P_L - C_L) + (P_L - C_L)D_L$$

Before proceeding further we make the following simplifying assumption: Assumption 1 (A1): $0 < B < \frac{7}{16}$

This assumption is entirely reasonable as it simply says that those consumers who do not recognize the innovation and do not return to the innovator is at least fractionally less than half of the total consumers. Now,

$$\Pi_L = (P_L - C_L)\left[\frac{B[(H-L)-(P_H-P_L)]}{H-L} + \frac{P_H-P_L}{H-L}\right]$$

$$\frac{\partial \Pi_L}{\partial P_L} = B\frac{(H-L)-(P_H-P_L)}{H-L} + \frac{P_H-P_L}{H-L} + (P_L-C_L)\frac{(B-1)}{H-L} = 0$$

$$P_L = \frac{C_L+P_H}{2} - \frac{B(H-L)}{2(B-1)} \tag{7}$$

Substitute equation (7) into equation (6)

$$\begin{aligned} P_H &= \frac{H-L+C_H}{2} + \frac{C_L+P_H}{4} - \frac{B(H-L)}{4(B-1)} \\ &= \frac{(H-L)+(2C_H+C_L)}{3} + \frac{(H-L)}{3(1-B)} > 0 \end{aligned} \tag{8}$$

Since $1-B>0$ and $H-L>0$

Substitute equation (8) into equation (7)

$$\begin{aligned} P_L &= \frac{C_L}{2} + \frac{H-L+2C_H+C_L}{6} + \frac{H-L}{6(1-B)} - \frac{B(H-L)}{2(B-1)} \\ &= \frac{H-L+C_H+2C_L}{3} - \frac{B(H-L)}{3(1-B)} > 0 \end{aligned} \tag{9}$$

Note[7] :

$$\frac{B}{1-B} < 1 \Rightarrow \frac{B}{1-B}\frac{(H-L)}{3} < \frac{(H-L)}{3}$$
$$\Rightarrow \frac{H-L+C_H+2C_L}{3} - \frac{B(H-L)}{3(1-B)} > 0$$

The following propositions then follow naturally:

Proposition 1: The innovator will necessarily charge a higher price than the non-innovator.

Proposition 2: As one firm innovates, or as products become more distinct, each firm will be able to charge a higher price.

Proposition 3: The wedge between the price of the innovator and the non-innovator is an increasing function of the difference between product types.

Proof of Propositions

Proposition 1

$$P_H = \frac{(H-L)+(2C_H+C_L)}{3} + \frac{(H-L)}{3(1-B)}$$

$$P_L = \frac{H-L+C_H+2C_L}{3} - \frac{B(H-L)}{3(1-B)}$$

$$(P_H - P_L) = \left[\frac{(H-L)+(2C_H+C_L)}{3} + \frac{(H-L)}{3(B-1)}\right] - \left[\frac{H-L+C_H+2C_L}{3} - \frac{B(H-L)}{3(1-B)}\right]$$

$$= \frac{C_H-C_L}{3} + \frac{(H-L)(1+B)}{3(1-B)} > 0 \quad (10)$$

Proposition 2

$$\frac{\partial P_H}{\partial(H-L)} = \frac{1}{3} + \frac{1}{3(1-B)} \geq 0;$$
$$\frac{\partial P_L}{\partial(H-L)} = \frac{1}{3} - \frac{B}{3(1-B)} \geq 0$$

[7] This is true by Assumption 1.

This follows from Assumption 1, which specifically says $B < \frac{1}{2}$, which then implies $\frac{\partial P_L}{\partial(H-L)} \geq 0$. That is, the more distinct products become the more flexibility the non-innovator gains over the price charged.

Proposition 3

$$\frac{\partial(P_H - P_L)}{\partial(H-L)} = \frac{1+B}{3(1-B)} > 0$$

This completes the solution for the second stage of the game. We can now substitute the price functions into the profit functions to get profits entirely as functions of type. Writing the first stage profit functions, we have:

High Type Firm

$$\Pi_H = \left[\frac{(H-L)+C_L-C_H}{3} + \frac{H-L}{3(1-B)}\right]\left[\frac{3(H-L)+C_L-C_H}{3(H-L)} - \frac{(1+B)}{3(1-B)}\right]$$

Low Type Firm

$$\Pi_L = \left[\frac{H-L+C_L-C_H}{3} + \frac{B(H-L)}{3(1-B)}\right]\left[\frac{(1+B)}{H-L}\right]\left[\frac{(H-L)+C_L-C_H}{3} + \frac{(H-L)(1+B)}{3(1-B)}\right]$$

These profit functions lend themselves to explicit solutions for the parameters chosen in the first stage of the game and are consistent with the literature on maximal product differentiation. However, a complete analysis of the game depends on how firms' profits react to changes in quality and the fraction of persons who recognize the innovation immediately. In addition, the analysis of the game depends on how the firms' expected profits react to the expected mean rate of arrival of the consumers who did not recognize the innovation immediately. The use of comparative statics is an elegant and instructive way to conduct the analysis (Thomas, 2010). This is done in the following section.

3 Comparative Statics

In this section, we assume that there are no fixed costs. Thus in the analysis that follows, marginal costs are captured as a function of type and is strictly increasing in innovation for the innovator. We also assume that marginal costs are strictly convex. Types (innovated product and non innovated product) are exogenous variables as we are working with profits in the second stage of the game. The equilibrium outcome is such that $V = \frac{P_H - P_L}{H-L}$ which is greater than or equal to zero. Under (A1) and the *No Leap Frogging Result* which will be shown later $C'_H < 1/9$. Furthermore, a second assumption, Assumption 2 (A2), is made, and under these conditions we are be able to analyze market reactions to innovation. Now, let $\Delta C \equiv C_H - C_L$ and $\Delta t \equiv H - L$ and consider (A2), which is as follows:

$$\max\left\{\frac{2C'_L-1}{3}+\frac{B}{3(1-B)},\frac{3C'_H}{3C'_H+1}\right\}<V<\frac{(1-C'_L)(1-B)+1}{3(1-B)} \tag{11}$$

From A2 and the fact that both $C'_L, C'_H > 0$ it follows naturally that:

$$V<\frac{(1-C'_L)(1-B)+1}{3(1-B)}<\frac{(1+2C'_H)(1-B)+1}{3(1-B)}$$

The inequality immediately above will be used to show a specific result and thus we present it here. The boundaries which appear in equation 11 are used in the proofs below and are therefore explicitly stated for clarity. The assumption A2 ensures that no firm's market share becomes too large or too small. This will reduce or eliminate any one firm's ability to monopolize the market. Thomas (2011) states that this is the strategic effect and it dominates just as in the textbook model where maximal product differentiation is sustained.

Theorem 1: *Under assumption* $(A2)$*, the principle of maximal product differentiation is sustained.*

The proof of theorem 1 is broken down into four propositions, propositions (4),(5),(6) and (7). The proof of these propositions are shown in equations 12 through 24 and demonstrate that firms have an incentive to encourage their rivals to distinguish themselves and that firms' profitability are increasing in the distance between types. In addition, proposition (6) and (7) show that profits are increasing in the number of persons who recognize the innovation immediately and expected profits are increasing in the expected mean rate of arrival of customers who return to the innovator. The optimal result is consistent with that of maximal product differentiation.

Theorem 2: *Under* $(A2)$*, it is impossible for the non-innovator to leap frog or mimic the innovator and reap the benefits of being an innovator.*

We will first prove *Theorem 2* as its results are used in the proofs related to *Theorem 1.*

Proof of Theorem 2:

Guaranteeing that $V\epsilon(0,1)$ while $(A2)$ is sustained has the following implication: *The innovator must be more efficient in innovating than the non-innovator.* Observing $(A2)$ in relation to $V\epsilon(0,1)$ requires that[8]:

[8] The second term in max{} in (A2) is unambiguously ¿ 0 and therefore needs not be considered here.

$$\frac{2C'_L - 1}{3} + \frac{B}{3(1-B)} > 0 \Leftrightarrow C'_L > \frac{1}{2} - \frac{B}{2(1-B)}$$

This requirement ensures that V is greater than zero. Additionally, considering that $(A2)$ is satisfied for $V < \frac{(1+2C'_H)(1-B)+1}{3(1-B)}$ but less than 1, requires that[9]:

$C'_H < \frac{1}{2} - \frac{B}{2(1-B)} \Rightarrow C'_H < \frac{1}{2} - \frac{B}{2(1-B)} < C'_L$. That is $C'_H < C'_L$.

This says that the innovator is more efficient in innovation activities. This could be due to the fact that the innovator has more highly skilled workers and advanced machines, etc. than the non-innovator. This helps to explain why firms choose to innovate or not to innovate. With the view in mind that $C'_L > C'_H$ the non-innovator will not choose to innovate. In order for the non-innovator to make positive profits, given its greater rate of increase of marginal cost, the innovator would be in a position to undercut the non-innovator which tries to innovate. If the non- innovator chooses to innovate then both firms would again be producing a homogenous product but the non-innovator would be charging a higher price thus passing through more cost to the consumers. This means that the non-innovator will face zero demand and thus gain zero revenues. With these costs, the non-innovator will choose not to innovate, thus cost pass through is an impediment to innovation for the non-innovator. On the other hand, cost pass through is not an impediment to innovation for the innovator as we will now state and show.

Proposition 4: *Under assumption $(A2)$, there is an incentive for firms to encourage their rivals to distinguish themselves*[10].

Proposition 5: *Under $(A2)$, there is a unique subgame nash equilibrium where firms choose different types.*

Proposition 6: *The innovator's profit is increasing in the number of consumers who recognize the innovation immediately (Z), while the non-innovator's profits is decreasing in Z.*

Proposition 7: *The innovator's expected profit is increasing in the expected mean rate of arrival of the consumers who did not recognize the innovation immediately (λ), while the non-innovator's profit is decreasing in (λ).*

Proof of Proposition 4

Low Type Firm

[9]From this result and (A1), $B < 7/16$, it is easy to show that $C'_H < 1/9$.

[10]Note: This is reflected in equations 12 and 13 showing that firms' profitability is increasing in the distance between types.

$$\frac{\partial \Pi_L}{\partial H} = [P_L - C_L]\left[\frac{\partial D_L}{\partial H} + \frac{\partial D_L}{\partial P_H}.\frac{\partial P_H}{\partial H}\right]$$

By the envelope theorem we need not take derivative w.r.t. P_L, furthermore

$$\frac{\partial D_L}{\partial H} = -\frac{(P_H - P_L)}{(H-L)^2} = -\frac{V}{H-L}; \frac{\partial D_L}{\partial P_H} = \frac{1}{H-L}$$

$$\frac{\partial P_H}{\partial H} = \frac{1+2C'(H)}{3} + \frac{1}{3(1-B)}$$

Recall $D_L = V$, therefore,

$$\begin{aligned} \frac{\partial \Pi_L}{\partial H} &= [P_L - C_L]\left[\frac{-V}{H-L} + \frac{-1}{H-L}\left(\frac{1+2C'(H)}{3} + \frac{1}{3(1-B)}\right)\right] \\ &= [P_L - C_L]\left[\frac{-V}{H-L} + \frac{1+2C'(H)}{3(H-L)} + \frac{1}{3(1-B)(H-L)}\right] \quad (12) \end{aligned}$$

We want to show that as one firm innovates the other firm's (non-innovator's) profit increases. This shows that maximal product differentiation is beneficial to both firms. That is, both firms will try to convince the other to maximally differentiate itself. We need to show that equation 12 is greater than zero. We know that $P_L - C_L$ is positive, therefore for the entire equation to be positive, the term in square bracket must also be positive.

$$\begin{aligned} &\frac{-V}{H-L} + \frac{1+2C'(H)}{3(H-L)} + \frac{1}{3(1-B)(H-l)} > 0 \\ \Longleftrightarrow &\frac{-3(1-B)V + (1-B)(1+2C'H) + 1}{3(H-L)(1-B)} > 0 \\ &\Longleftrightarrow (1-B)(1+2C'H) + 1 > 3(1-B)V \\ &\Longleftrightarrow \frac{(1-B)(1+2C'H)+1}{3(1-B)} > V \\ &\Longleftrightarrow \frac{(1+2C'H)+1}{3} + \frac{1}{3(1-B)} > V \\ &\Longleftrightarrow V < \frac{1+2C'_H}{3} + \frac{1}{3(1-B)} \end{aligned}$$

The High Type Firm

$$\frac{\partial \Pi_H}{\partial L} = [P_H - C_H]\left[\frac{\partial D_H}{\partial L} + \frac{\partial D_H}{\partial P_L}.\frac{\partial P_L}{\partial L}\right]$$

By the envelope theorem we do not need to take derivative w.r.t. P_H, furthermore, recall $D_H = 1 - V$

$$\frac{\partial D_H}{\partial L} = -\frac{(P_H - P_L)}{(H-L)^2} = -\frac{V}{H-L}; \frac{\partial D_H}{\partial P_L} = \frac{1}{H-L}$$

$$\frac{\partial P_L}{\partial L} = \frac{-1+2C'(L)}{3} + \frac{B}{3(1-B)}$$

Now,

$$\begin{aligned}\frac{\partial \Pi_H}{\partial L} &= [-(P_H - C_H)]\left[\frac{V}{H-L} - \frac{1}{H-L}\left(\frac{2C'_L - 1}{3} + \frac{B}{3(1-B)}\right)\right] \\ &= -(P_H - C_H)\left[\frac{V}{H-L} - \frac{(2C'_L - 1)}{3(H-L)} - \frac{B}{3(1-B)(H-L)}\right] \quad (13)\end{aligned}$$

The maximal product differentiation result requires that if the non-innovator starts to innovate then its profits should decrease. Therefore, we need equation 13 to be negative. As a result we require that the following equation be positive.

$$(P_H - C_H)\left[\frac{3(1-B)V - (2C'_L - 1)(1-B) - B}{3(H-L)(1-B)}\right] > 0$$

$$\Leftrightarrow 3(1-B)V - (1-B)(2C'_L - 1) - B > 0$$

$$\Leftrightarrow 3(1-B)V > (1-B)(2C'_L - 1) + B$$

$$\Leftrightarrow V > \frac{(1-B)(2C'_L - 1) + B}{3(1-B)}$$

$$\Leftrightarrow V > \frac{2C'_L - 1}{3} + \frac{B}{3(1-B)}$$

$$\Leftrightarrow V > \frac{2C'_L - 1}{3} + \frac{B}{3(1-B)}$$

Proof of Proposition 5

The Low Type Firm

$$\frac{\partial \Pi_L}{\partial L} = [P_L - C_L]\left[\frac{\partial D_L}{\partial L} + \frac{\partial D_L}{\partial P_H}.\frac{\partial P_H}{\partial L}\right] - C'_L V$$

By the envelope theorem we need not take derivative w.r.t. P_L, furthermore. Recall $D_L = V$

$$\frac{\partial D_L}{\partial L} = \frac{(P_H - P_L)}{(H-L)^2} = \frac{V}{H-L}; \frac{\partial D_L}{\partial P_H} = \frac{1}{H-L}$$

$$\frac{\partial P_H}{\partial L} = \frac{-1 + C'(L)}{3} - \frac{1}{3(1-B)} = \frac{C'(L) - 1}{3} - \frac{1}{3(1-B)}$$

$$\frac{\partial \Pi_L}{\partial L} = [P_L - C_L]\left[\frac{V}{H-L} + \frac{1}{H-L}\left(\frac{C'_L - 1}{3} - \frac{1}{3(1-B)}\right)\right] - C'_L V$$

$$= [P_L - C_L]\left[\frac{V}{H-L} + \frac{(C'L - 1)}{3(H-L)} - \frac{1}{3(1-B)(H-L)}\right] - C'_L V \quad (14)$$

For maximal product differentiation we require that equation 14 be less than zero. Given that $P_L - C_L$ is positive, we need the term in the square brackets to be negative. That is, we need:

$$\left[\frac{3(1-B)V + (C'_L - 1)(1-B) - 1}{3(1-B)(H-L)}\right] < 0$$

Since the denominator is positive, we simply need:

$$\Leftrightarrow 3(1-B)V + (C'_L - 1)(1-B) - 1 < 0$$

$$\Leftrightarrow 3(1-B)V < 1 - (C'_L - 1)(1-B)$$

$$\Leftrightarrow V < \frac{1-(C'_L-1)(1-B)}{3(1-B)}$$

$$\Leftrightarrow V < \frac{1}{3(1-B)} + \frac{(1-C'_L)}{3}$$

$$\Leftrightarrow V < \frac{1-C'_L}{3} + \frac{1}{3(1-B)}$$

The High Type Firm

$$\frac{\partial \Pi_H}{\partial H} = [P_H - C_H]\left[\frac{\partial D_H}{\partial H} + \frac{\partial D_H}{\partial P_L}.\frac{\partial P_L}{\partial H}\right] - C'_H(1-V)$$

By the envelope theorem we need not take the derivative w.r.t. P_H

$$\frac{\partial D_H}{\partial H} = \frac{(P_H-P_L)}{(H-L)^2} = \frac{V}{H-L}; \frac{\partial D_H}{\partial P_L} = \frac{1}{H-L}$$

$$\frac{\partial P_L}{\partial H} = \frac{1+C'(H)}{3} - \frac{B}{3(1-B)}$$

Now,

$$\begin{aligned}
\frac{\partial \Pi_H}{\partial H} &= [(P_H-C_H)]\left[\frac{V}{H-L} + \frac{1}{H-L}\left(\frac{1+C'_H}{3} - \frac{B}{3(1-B)}\right) - C'_H(1-V)\right] \\
&= [(P_H-C_H)]\left[\frac{V}{H-L} + \frac{(1+C'_H)(1-B)-B}{3(H-L)(1-B)} - \frac{B}{3(1-B)(H-L)}\right] - C'_H(1-V) \\
&= [(P_H-C_H)]\left[\frac{3V(1-B)+1-2B+C'_H-C'_HB}{3(H-L)(1-B)}\right] - C'_H(1-V) > 0 \qquad (15)
\end{aligned}$$

For maximal product differentiation, we need equation 15 to be greater than zero. Substituting for $P_H - C_H$, yields the following result:

$$\left[\frac{H-L+C_L-C_H}{3} + \frac{H-L}{3(1-B)}\right]\left[\frac{3V(1-B)+1-2B+C'_H(1-B)}{3(H-L)(1-B)}\right] - C'_H(1-V) > 0$$

$$\Leftrightarrow \left[\frac{(\Delta t - \Delta C)(1-B)+(\Delta t)}{3(1-B)}\right]\left[\frac{3V(1-B)+1-2B+C'_H(1-B)}{3\Delta t(1-B)}\right] - C'_H(1-V) > 0$$

$$\Leftrightarrow \Delta t(1-B)\left[\frac{1+\frac{-\Delta C}{\Delta t}+\frac{1}{1-B}}{3(1-B)}\right]\left[\frac{3V(1-B)+1-2B+C'_H(1-B)}{3(\Delta t)(1-B)}\right]-C'_H(1-V)>0$$

$$\Leftrightarrow \frac{\left[1+\frac{-\Delta C}{\Delta t}+\frac{1}{1-B}\right]\left[3V(1-B)+1-2B+C'_H(1-B)\right]-9C'_H(1-V)(1-B)}{9(1-B)}>0$$

$$\Leftrightarrow \left[1+\frac{1}{1-B}-\frac{\Delta C}{\Delta t}\right]\left[3V+\frac{1-2B}{1-B}+C'_H\right]>9C'_H(1-V)$$

$$\Leftrightarrow \left[1+\frac{1}{1-B}-\frac{\Delta C}{\Delta t}\right]\frac{\left[3V+\frac{1-2B}{1-B}+C'_H\right]}{9C'_H}>1-V$$

$$\Leftrightarrow \left[1+\frac{1}{1-B}-\frac{\Delta C}{\Delta t}\right]\left[\frac{V}{3C'_H}+\frac{1}{9C'_H}-\frac{B}{9C'_H(1-B)}+\frac{1}{9}\right]>1-V$$

$$\Leftrightarrow V>1-\left[1+\frac{1}{1-B}-\frac{\Delta C}{\Delta t}\right]\left[\frac{V}{3C'_H}+\frac{1}{9C'_H}-\frac{B}{9C'_H(1-B)}+\frac{1}{9}\right]$$

$$\Leftrightarrow V>1-\left[\frac{\Delta t(1-B)+\Delta t-\Delta C(1-B)}{\Delta t(1-B)}\right]\left[\frac{3(1-B)V+1-2B+C'_H(1-B)}{9C'_H(1-B)}\right]$$

$$\Leftrightarrow V>\frac{9\Delta tC'_H(1-B)^2-\left[(\Delta t-\Delta C)(1-B)+\Delta t\right]\left[3(1-B)V+(1+C'_H)(1-B)-B\right]}{9C'_H\Delta t(1-B)^2}$$

$$\Leftrightarrow 9C'_H\Delta t(1-B)^2V+\left[(\Delta t-\Delta C)(1-B)+\Delta t\right]3(1-B)V>9\Delta tC'_H(1-B)^2-$$

$$\left[(\Delta t-\Delta C)(1-B)+\Delta t\right]\left[(1+C'_H)(1-B)-B\right]$$

$$\Leftrightarrow 9C'_H\Delta t(1-B)V+\left[(\Delta t-\Delta C)(1-B)+\Delta t\right]3V>\Delta t9C'_H(1-B)-$$

$$\left[(\Delta t - \Delta C)(1-B) + \Delta t\right]\left[(1 + C'_H) - \frac{B}{1-B}\right]$$

$$\Leftrightarrow V > \frac{9\Delta t C'_H(1-B) - \left[(\Delta t - \Delta C)(1-B) + \Delta t\right]\left[(1 + C'_H) - \frac{B}{1-B}\right]}{3[(\Delta t - \Delta C)(1-B) + \Delta t] + 9C'_H \Delta t(1-B)}$$

$$\Leftrightarrow V > \frac{9\Delta t C'_H - \left[(\Delta t - \Delta C) + \frac{\Delta t}{(1-B)}\right]\left[1 + C'_H - \frac{B}{1-B}\right]}{3[(\Delta t - \Delta C) + \frac{\Delta t}{1-B} + 3C'_H \Delta t]}$$

$$\Leftrightarrow V > \frac{9\Delta t C'_H - \left[(\Delta t - \Delta C) + \frac{\Delta t}{1-B}\right]\left[1 + C'_H - \frac{B}{1-B}\right]}{3[(\Delta t - \Delta C) + (3C'_H + \frac{1}{1-B})\Delta t]}$$

Provided that[11] $C'_H < 1/9$ the right hand side of the above inequality is unambiguously negative and since $0 < V < 1$ the inequality is upheld. In order to analyze cost pass through we take the following alternate approach: For $\frac{\partial \Pi_H}{\partial H} > 0$ we need the following:

$$\left[1 - \frac{\Delta C}{H-L} + \frac{1}{1-B}\right]\left[3V(1-B) + 1 - 2B + C'_H(1-B)\right] - 9C'_H(1-V)(1-B) > 0$$

Expanding parentheses, if any one of the positive terms is greater than everything that follows the minus sign then we can conclude that $\frac{\partial \Pi_H}{\partial H} > 0$. We therefore proceed as follows:

$$\left[1 + \frac{C_L - C_H}{H-L} + \frac{1}{1-B}\right]\left[3V(1-B) + 1 - 2B + C'_H(1-B)\right] - 9C'_H(1-V)(1-B) > 0 \quad (16)$$

We will multiply the first term of each parenthesis and set the result greater than $9C'_H(1-V)(1-B)$. Thus:

$$(1)\left[3V(1-B)\right] > 9C'_H(1-V)(1-B)$$

[11] As a reminder, this result is a consequence of (A1) and the No Leap Frogging Result, Theorem 2.

$$\Leftrightarrow 3V(1-B) > 9C'_H(1-V)(1-B)$$

$$\Leftrightarrow 3V > 9C'_H(1-V)$$

$$\Leftrightarrow V > 3C'_H(1-V)$$

$$\Leftrightarrow V > 3C'_H - 3C'_H V$$

$$\Leftrightarrow V(1+3C'_H) > 3C'_H$$

$$\Leftrightarrow V > \frac{3C'_H}{3C'_H+1} \tag{17}$$

but $V = \frac{P_H - P_L}{H-L} = \frac{\Delta P}{\Delta t}$

$$\frac{\Delta P}{\Delta t} > \frac{3C'_H}{3C'_H+1}$$

$$\Delta P > \frac{3C'_H}{3C'_H+1}\Delta t$$

$$P_H - P_L > \frac{3C'_H}{3C'_H+1}(H-L)$$

Of course, $C'_H < 1$ and thus: $3C'_H + 1 < 4 \Rightarrow P_H - P_L > \frac{3C'_H}{4}(H-L)$. This says that under maximal differentiation, where $H = 1$ *and* $L = 0$, at least 3/4 or 75 % of the change in cost attributed to innovation is passed on to consumers. That is:

$$P_H - P_L > \frac{3C'_H}{3C'_H+1}(H-L) > \frac{3C'_H}{4}(H-L) = \frac{3}{4}C'_H(H-L)$$

This says that the change in price must be greater than $\frac{3}{4}$ of the cost change. This change in price or $P_H - P_L$ is precisely their additional cost that is passed on to the consumers. The price the innovator charges is

$$P_H > P_L + \frac{3}{4}C'_H(H - L)$$

This implies that the price the innovator charges is increasing in the innovation cost. It also implies that as products become more distinct, the innovator like the non-innovator has more flexibility over the price he charges. If there is maximal vertical product differentiation (that is H=1 and L=0) then we find that

$$P_H > P_L + \frac{3}{4}C'_H$$

This means that in the presence of maximal vertical product differentiation, the innovator will pass through a minimum of 75 percent of the cost of innovation to the consumer.
More generally, under Assumption 2, propositions 4 and 5 demonstrate that the results of maximal differentiation holds with variable marginal costs and when consumers are allowed to switch between the innovated and non innovated products. The propositions also show that as the innovating firm distinguishes its product from the non-innovating firm, both firms' profits increase.

Proof of Proposition 6

In order to simplify the differentiation for this section, the price functions are rewritten as follows:

$$P_H = \frac{H - L + 2C_H + C_L}{3} - \frac{H - L}{3[Z + (1 - Z)\lambda]}$$

$$P_L = \frac{H - L + C_H + 2C_L}{3} - \frac{(H - L)(1 - Z)(1 - \lambda)}{3[Z + (1 - Z)\lambda]}$$

The Low Type Firm

$$\frac{\partial \Pi_L}{\partial Z} = [P_L - C_L]\Bigg[(1 - Z)(1 - \lambda)\left(\frac{\partial D_H}{\partial Z} + \frac{\partial D_H}{\partial P_H}.\frac{\partial P_H}{\partial Z}\right)$$
$$-(1 - \lambda)D_H + \left(\frac{\partial D_L}{\partial Z} + \frac{\partial D_L}{\partial P_H}.\frac{\partial P_H}{\partial Z}\right)\Bigg]$$

By the envelope theorem we need not take the derivative w.r.t. P_L, Furthermore,

$$D_L = \frac{P_H - P_L}{H - L} = V$$

$$\frac{\partial D_L}{\partial Z} = \frac{-(1-\lambda)(H-L)}{3[\lambda+(1-\lambda)Z]^2} - \frac{(H-L)(1-\lambda)}{3[\lambda+(1-\lambda)Z]^2(H-L)} = \frac{-2(1-\lambda)}{3[\lambda+(1-\lambda)Z]^2}$$

$$\frac{\partial D_L}{\partial P_H} = \frac{1}{H-L}; \frac{\partial P_H}{\partial L} = \frac{-(1-\lambda)(H-L)}{3[\lambda+(1-\lambda)Z]^2}$$

$$\begin{aligned}
\frac{\partial \Pi_L}{\partial Z} &= (P_L - C_L)\Bigg[(1-Z)(1-\lambda)\left(\frac{1-\lambda}{[\lambda+(1-\lambda)Z]^2}\right) \\
&- (1-\lambda)(1-V) - \frac{-2(1-\lambda)}{3[\lambda+(1-\lambda)Z]^2} - \frac{1}{H-L}\left(\frac{(1-\lambda)(H-L)}{3[\lambda+(1-\lambda)Z]^2}\right)\Bigg] \\
&= (P_L - C_L)\left[(1-Z)(1-\lambda)\left(\frac{1-\lambda}{[\lambda+(1-\lambda)Z]^2}\right) - (1-\lambda)(1-V) - \frac{(1-\lambda)}{[\lambda+(1-\lambda)Z]^2}\right] \\
&= (P_L - C_L)\left[\frac{1-\lambda}{[\lambda+(1-\lambda)Z]^2}\Big((1-Z)(1-\lambda)-1\Big) - (1-\lambda)(1-V)\right] < 0 \qquad (18)
\end{aligned}$$

This shows that the non-innovator's profit function is decreasing in the number of persons who recognize the innovation immediately.

<u>The High Type Firm</u>

$$\frac{\partial \Pi_H}{\partial Z} = [P_H - C_H]\left[\left(\frac{\partial D_H}{\partial Z} + \frac{\partial D_H}{\partial P_L}.\frac{\partial P_L}{\partial Z}\right)\Big(Z+(1-Z)\lambda\Big) + D_H - D_H\lambda\right]$$

By the envelope theorem we need not take the derivative w.r.t. P_H, Furthermore,

For $\frac{\partial \Pi_H}{\partial Z}$ we need:

For simplification, the prices are rewritten as follows:

$$P_H = \frac{H-L+2C_H+C_L}{3} + \frac{H-L}{3[\lambda+(1-\lambda)Z]} \qquad (19)$$

$$P_L = \frac{H-L+C_H+2C_L}{3} + \frac{(H-L)(1-2)(1-\lambda)}{3[\lambda+(1-\lambda)Z]} \qquad (20)$$

$$\frac{\partial D_H}{\partial Z} = \frac{2(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2}; \frac{\partial D_H}{\partial P_L} = \frac{1}{H-L}$$

$$\frac{\partial P_L}{\partial Z} = \frac{3[\lambda + (1-\lambda)Z](1-\lambda)(H-L) + (H-L)(1-Z)(1-\lambda)[3(1-\lambda)]}{9[\lambda + (1-\lambda)Z]^2}$$

$$= \frac{(H-L)(1-\lambda)\Bigg[\lambda + (1-\lambda)Z + (1-Z)(1-\lambda)\Bigg]}{3[\lambda + (1-\lambda)Z]^2}$$

$$= \frac{(H-L)(1-\lambda)\Bigg[\lambda + (1-\lambda)[Z + (1-Z)]\Bigg]}{3[\lambda + (1-\lambda)Z]^2}$$

$$= \frac{(H-L)(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2}$$

Since $Z + (1-Z) = 1$ and $\lambda + (1-\lambda) = 1$

$$\frac{\partial D_H}{\partial Z} = -\Bigg[-\frac{(H-L)(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2} - \frac{(H-L)(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2}\Bigg]\frac{1}{H-L} = \frac{2(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2}$$

Now,

$$\begin{aligned}\frac{\partial \Pi_H}{\partial Z} &= (P_H - C_H)\Bigg[\frac{2(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2} + \frac{1}{H-L}\Bigg(\frac{(H-L)(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2}[Z + (1-Z)\lambda] \\ &+ D_H - D_H\lambda\Bigg)\Bigg] \\ &= (P_H - C_H)\Bigg[\frac{3(1-\lambda)}{3[\lambda + (1-\lambda)Z]^2}[Z + (1-Z)\lambda] + 1 - V - (1-V)\lambda\Bigg] \\ &= (P_H - C_H)\Bigg[\frac{(1-\lambda)[Z + (1-Z)\lambda]}{[\lambda + (1-\lambda)Z]^2} + (1-V)(1-\lambda)\Bigg] > 0 \qquad (21)\end{aligned}$$

Therefore, the innovator's profit is increasing in the number of persons who recognize the innovation immediately.

Proof of Proposition 7: ***The Effect of the Mean Rate of Arrival of Returning Customers:***

The Low Type Firm

$$\frac{\partial \Pi_L}{\partial \lambda} = [P_L - C_L]\left[(1-Z)(1-\lambda)\left(\frac{\partial D_H}{\partial \lambda} + \frac{\partial D_H}{\partial P_H}.\frac{\partial P_H}{\partial \lambda}\right)\right] - (1-Z)D_H + \left(\frac{\partial D_L}{\partial \lambda} + \frac{\partial D_L}{\partial P_H}.\frac{\partial P_H}{\partial \lambda}\right)$$

By the envelope theorem we need not take the derivative w.r.t. P_L

$$\frac{\partial P_H}{\partial \lambda} = -\frac{(H-L)(1-Z)}{3[Z+(1-Z)\lambda]^2}$$

$$\frac{\partial D_H}{\partial \lambda} = \frac{2(1-Z)}{3[z+(1-z)\lambda]^2}; \frac{\partial D_H}{\partial P_H} = \frac{-1}{H-L}; \frac{\partial P_H}{\partial \lambda} = \frac{-(1-Z)(H-L)}{3[z+(1-z)\lambda]^2}$$

$$\frac{\partial P_L}{\partial \lambda} = \frac{(H-L)(1-Z)}{3[z+(1-z)\lambda]^2}; \frac{\partial D_L}{\partial P_H} = \frac{1}{H-L}; \frac{\partial D_L}{\partial \lambda} = \frac{-2(1-Z)}{3[z+(1-z)\lambda]^2}$$

Now,

$$\begin{aligned} \frac{\partial \pi_L}{\partial \lambda} &= [P_L - C_L]\left[(1-Z)(1-\lambda)\left(\frac{2(1-Z)}{3[z+(1-z)\lambda]^2} + \frac{(1-Z)}{3[z+(1-z)\lambda]^2}\right)\right. \\ &- (1-V)(1-Z) + \left.\left[\left(\frac{-2(1-Z)}{3[z+(1-z)\lambda]^2}\right) - \frac{1}{H-L}\left(\frac{(H-L)(1-Z)}{3[z+(1-z)\lambda]^2}\right)\right]\right] \\ &= [P_L - C_L]\left[\left(\frac{(1-Z)}{[z+(1-z)\lambda]^2}\right)\left((1-Z)(1-\lambda) - 1\right) - (1-Z)(1-V)\right] < 0 \quad (22) \end{aligned}$$

Thus, the non-innovator's expected profits is decreasing in the expected number of customers who recognize the innovation at a later date and return to the firm that innovates.

High Type Firm

$$\frac{\partial \Pi_H}{\partial \lambda} = [P_H - C_H]\left[\frac{\partial D_H}{\partial \lambda} + \frac{\partial D_H}{\partial P_L}.\frac{\partial P_L}{\partial \lambda}\right][Z+(1-Z)\lambda] + (1-Z)D_H \quad (23)$$

By the envelope theorem we need not take the derivative w.r.t. P_H

$$\frac{\partial D_H}{\partial P_L} = \frac{1}{H-L}; \frac{\partial D_H}{\partial P_L}.\frac{\partial P_L}{\partial \lambda} = \frac{(1-Z)}{3[Z+(1-Z)\lambda]^2}$$

$$\frac{\partial D_H}{\partial \lambda} = \frac{1}{H-L} = \frac{\frac{-\partial P_H}{\partial P_L} + \frac{\partial P_L}{\partial \lambda}}{H-L} = \frac{\frac{(H-L)(1-Z)+(H-L)(1-Z)}{3[Z+(1-Z)\lambda]^2}}{H-L} = \frac{2(1-Z)}{3[Z+(1-Z)\lambda]^2}$$

$$\frac{\frac{\partial P_L}{\partial \lambda} = 3[Z+(1-Z)\lambda][-(H-L)(1-Z)(-1)] - [-(H-L)(1-Z)(1-\lambda)3(1-Z)]}{9[z+(1-Z)\lambda]^2}$$

$$= \frac{(H-L)(1-Z)[z+(1-z)\lambda] + (H-L)(1-Z)^2(1-\lambda)}{3[z+(1-z)\lambda]^2}$$

$$= \frac{(H-L)(1-Z)[z+(1-z)\lambda] + (1-Z)(1-\lambda)}{3[z+(1-z)\lambda]^2} = \frac{(H-L)(1-Z)[z+(1-z)(1)]}{3[z+(1-z)\lambda]^2}$$

$$= \frac{(H-L)(1-Z)}{3[z+(1-z)\lambda]^2}$$

Now,

$$\frac{\partial \Pi_H}{\partial \lambda} = [P_H - C_H]\left[\frac{2(1-Z)}{3[z+(1-z)\lambda]^2} + \frac{1}{H-L}\left(\frac{(H-L)(1-Z)}{3[Z+(1-Z)\lambda]^2}\right)[Z+(1-Z)\lambda] + (1-Z)(1-V)\right]$$

$$\frac{\partial \Pi_H}{\partial \lambda} = [P_H - C_H]\left[\frac{1-Z}{[Z+(1-Z)\lambda]} + (1-Z)(1-V)\right] > 0 \qquad (24)$$

This of course says that the innovator's profits are increasing in the mean rate of arrival of returning customers. More generally, Propositions 6 and 7 showed that the innovator's profit is increasing in the number of persons who recognize the innovation immediately and the expected rate of return of the consumers who switch to the non-innovator. Since $\frac{\partial \Pi_H}{\partial \lambda} > 0$ then Π_H and λ are positively related. Also, since $\frac{\partial \Pi_H}{\partial Z} > 0$ then Π_H and Z are positively related.

It is only natural that the impact of λ and Z reinforce each other. That is, the greater the fraction of persons who recognize the innovation immediately, the faster the rate of return of those who switched. This results as more persons will be able to spread the word that the new product is good. On the other hand, if the fraction of persons who realize the innovation immediately increases or if the rate of return is faster then the non-innovator's profits will decrease. More persons will buy the innovated product and the faster the customers who switched to the non-innovator will leave to buy the innovated product from the innovative firm.

In addition, Thomas (2010) states that if firms prefer to face high cost now and face lower cost later they will choose to innovate. On the other hand, firms that prefer to face low cost now but face higher cost for later improvements will choose not to innovate. A_2 is therefore quite plausible. Of course, (A2) is most simply expressed as $0 < V < 1$, which implies that both firms have positive market shares. That is, the market will not be monopolized by any one firm as both firms face positive demand. This also means that one firm will always choose not to innovate.

4 Conclusion

Under variable marginal cost structure; models of vertical product differentiation result in maximal product differentiation under reasonable conditions. This is in keeping with the findings of D'Aspermont Gabszewick and Thissue (1979). The model presented in this paper is an extension of that usually described, with an allowance to incorporate varying but different marginal costs across firms. We also allowed for the inclusion of consumers switching between firms with the possibility of returning to the innovator/firm from which the switch initially occurred. In this framework, we were able to provide sufficient conditions under which the results are sustained. We note, however, the need for empirical evidence to validate the results of the model. Provided that our assumptions hold, we find that both the innovator and non-innovator find it mutually beneficial for each firm to encourage the other to maximally differentiate its product.

Although under maximal product differentiation, competition is less intense, thus each firm can charge a higher price, consumers have a better range of goods to choose from. Also, we showed that under maximal product differentiation the non-innovator is less efficient at producing quality than the innovator. Consequently, the non-innovator will not pretend to be high type/innovative firm in order to reap the benefits of innovation. An implication of our model is that if the assumptions were violated then the results would be ambiguous. Again the need for validation by empirical analyses is imperative.

A result that was straight forward is that the higher the fraction of persons that recognize the innovation immediately the faster will switchers recognize the value of the innovation and return to the innovative firm. This result applies free of any and all restrictions. This means that the greater the fraction of consumers who recognize the innovation immediately, the greater the profits of the innovator. In contrast, if the fraction of consumers who recognize the innovation immediately increases then the profits of the non-innovator will decrease. The result for the expected mean rate of arrival are the same as the results for the fraction of consumers who recognize the innovation immediately. This is expected. If more persons recognize the innovation immediately the others should return at a faster rate. This may be due to "word of mouth" or formal advertisement by the innovator.

This model does not explicitly model switching cost. We should note however that the inclusion of switching cost in this model would simply reduce λ in the case of a poisson distribution and increase θ in the case of a exponential distribution.

We further found that in equilibrium, the innovator will pass through more than 75 percent of the cost changes to the consumers. Regardless of the fact that the innovative firm may not pass through all the costs of innovation to consumers, the innovative firm still maintains positive profits. That is innovation makes the innovative firm better-off. In concluding, we answer the question, Is cost pass through an impediment to innovation? Cost pass through is an impediment to innovation for the non-innovator (the low type or less efficient firm) but it is not an impediment to innovation for the innovator.

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6 Appendices

Appendix A: The First Order Conditions

$$\Pi_H(P_H, P_L) = Z(P_H - C_H)D_H + (1-Z)\lambda D_H(P_H, C_H)$$

$$= (P_H - C_H)D_H[Z + (1-Z)\lambda] \quad (25)$$

Now, let $A \equiv Z + (1-Z)\lambda \Rightarrow$

$$\Pi_H = A(P_H - C_H)\left[\frac{(H-L)-(P_H-P_L)}{H-L}\right]$$

$$= A\left[P_H\left(\frac{(H-L)-(P_H-P_L)}{H-L}\right) - C_H\left(\frac{(H-L)-(P_H-P_L)}{H-L}\right)\right] \quad (26)$$

$$\frac{\partial \Pi_H}{\partial P_H} = A\left[\frac{(H-L)-(P_H-P_L)}{H-L} + P_H\left(\frac{-1}{H-L}\right) - C_H\left(\frac{-1}{H-L}\right)\right]$$

$$= A\left[\frac{(H-L)-(P_H-P_L)}{H-L} + P_H\left(\frac{P_H}{H-L}\right) - C_H\left(\frac{C_H}{H-L}\right)\right]$$

$$= A\left[\frac{(H-L)-P_H+P_L-P_H+C_H)}{H-L}\right] \quad (27)$$

Divide through by A and multiply through by $H-L$ and set the result $= 0$, to get:

$$H - L + P_L + C_H - 2P_H = 0 \Longrightarrow P_H = \tfrac{(H-L)+P_L+C_H}{2} = \tfrac{H-L+C_H}{2} + \tfrac{P_L}{2}$$

Now consider:

$$\Pi_L = (1-Z)[1-\lambda]D_H(P_L - C_L) + (P_L - C_L)D_L \quad (28)$$

$$\Pi_L = (1-Z)[1-\lambda]\left[\frac{(H-L)-(P_H-P_L)}{H-L}\right](P_L-C_L)+(P_L-C_L)\left[\frac{P_H-P_L}{H-L}\right] \quad (29)$$

Now, let $B \equiv (1-Z)[1-\lambda]$, and consider Assumption 1: $0 < B < \frac{7}{16}$.

$$\Pi_L = (P_L-C_L)\left[\frac{B[(H-L)(P_H-P_L)]}{H-L}+\frac{P_H-P_L}{H-L}\right]$$

$$\Pi_L = P_L\left[\frac{B[(H-L)-(P_H-P_L)]}{H-L}+\frac{P_H-P_L}{H-L}\right]$$

$$- C_L\left[\frac{B[(H-L)-(P_H-P_L)]}{H-L}+\frac{P_H-P_L}{H-L}\right]$$

$$\frac{\partial\Pi_L}{\partial P_L} = \left[\frac{B(H-L)-(P_H-P_L)}{H-L}\right]+P_L\left[\frac{B(-1)}{H-L}-\frac{1}{H-L}\right]$$

$$- C_L\left[\frac{B(-1)}{H-L}-\frac{1}{H-L}\right] \Leftrightarrow$$

$$\frac{\partial\Pi_L}{\partial P_L} = B\frac{(H-L)-(P_H-P_L)}{H-L}+\frac{P_H-P_L}{H-L}+(P_L-C_L)\frac{(B-1)}{H-L}=0 \quad (30)$$

$$\Leftrightarrow B\frac{(H-L)-(P_H-P_L)}{H-L}+\frac{P_H-P_L}{H-L} = -(P_L-C_L)\frac{(B-1)}{H-L} \quad (31)$$

Multiply through by $H-L$, to get:

$$B[(H-L)-(P_H-P_L)]+P_H-P_L=-P_L(B-1)+C_L(B-1)$$

$$\Leftrightarrow B[(H-L)-BP_H+BP_L+P_H-P_L=-P_L(B-1)+C_L(B-1)$$

$$\Leftrightarrow B(H-L)+P_H(B-1)+2P_L(B-1)=C_L(B-1)$$

$$\Leftrightarrow 2P_L(B-1) = C_L(B-1) - P_H(B-1) - B(H-L)$$

$$\begin{aligned} \Leftrightarrow P_L &= C_L\frac{(B-1) - P_H(B-1) - B(H-L)}{2(B-1)} \\ &= \frac{(C_L + P_H)(B-1) - B(H-L)}{2(B-1)} \\ &= \frac{C_L + P_H}{2} - \frac{B(H-L)}{2(B-1)} \end{aligned} \tag{32}$$

Substitute into P_H to get:

$$\begin{aligned} P_H &= \frac{H - L + C_H}{2} + \frac{P_L}{2} \\ &= \frac{H - L + C_H}{2} + \frac{C_L + P_H}{4} - \frac{B(H-L)}{4(B-1)} \\ P_H - \frac{P_H}{4} &= \frac{H - L + C_H}{2} + \frac{C_L}{4} - \frac{B(H-L)}{4(B-1)} \\ \frac{3P_H}{4} &= \frac{(H-L) + C_H}{2} + \frac{C_L}{4} - \frac{B(H-L)}{4(B-1)} \\ P_H &= \frac{2(H-L) + 2C_H}{3} + \frac{C_L}{3} - \frac{B(H-L)}{4(B-1)} \\ P_H &= \frac{2(B-1)(H-L) + 2C_H(B-1) + C_L(B-1) - B(H-L)}{3(B-1)} \\ P_H &= \frac{(B-2)(H-L) + (B-1)(2C_H + C_L)}{3(B-1)} \\ P_H &= \frac{(H-L) + (2C_H + C_L)}{3} - \frac{(H-L)}{3(B-1)}; B - 1 < 0 \Rightarrow 1 - B > 0 \end{aligned}$$

$$P_H = \frac{(H-L)+(2C_H+C_L)}{3} + \frac{(H-L)}{3(1-B)} > 0; Since 1-B > 0 \quad (33)$$

Substitute into into equation 32, to get:

$$\begin{aligned}
P_L &= \frac{C_L}{2} + \frac{P_H}{2} - \frac{B(H-L)}{2(B-1)} \\
&= \frac{C_L}{2} + \frac{H-L+2C_H+C_L}{6} + \frac{H-L}{6(1-B)} - \frac{B(H-L)}{2(B-1)} \\
&= \frac{C_L}{2} + \frac{H-L+2C_H+C_L}{6} + \frac{H-L}{6(1-B)} + \frac{3B(H-L)}{6(B-1)} \\
&= \frac{3C_L+H-L+2C_H+C_L}{6} + \frac{(H-L)+3B(H-L)}{6(B-1)} \\
&= \frac{H-L+2C_H+4C_L}{6} + \frac{(H-L)(1-3B)}{6(1-B)} \\
&= \frac{(1-B)(H-L)+(1-B)(2C_H+4C_L)+(1-3B)(H-L)}{6(1-B)} \\
&= \frac{(2-4B)(H-L)}{6(1-B)} + \frac{(1-B)(2C_H+4C_L)}{6(1-B)} \\
&= \frac{(1-2B)(H-L)}{3(1-B)} + \frac{C_H+2C_L}{3} \\
&= \frac{(1-2B)(H-L)}{3(1-B)} - \frac{B(H-L)}{3(1-B)} + \frac{C_H+2C_L}{3} \\
&= \frac{H-L+C_H+2C_L}{3} - \frac{B(H-L)}{3(1-B)} > 0 \qquad (34)
\end{aligned}$$