Information Advantage of the Early Bird Entrepreneur

Kario-Paul Brown*
And
John Caillaud
University of Houston

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Abstract

This paper formalises a model of entrepreneurial opportunities with temporary asymmetric information. At the start date an entrepreneur will have some information not available to other entrepreneurs, but as entrepreneurs interact socially over time, information diffuses to other entrepreneurs in the economy. A scan of the literature shows the sparseness of contributions on the effects of the spread of information on entrepreneurial opportunities. To my knowledge, there is only one contribution that analyses the effects of the spread of information on entrepreneurial opportunities and that contribution focused on the effects of rumours on the price of securities[1]. This paper will add to existing knowledge by providing the analytical framework to assess the behaviour of the risk premium, as information diffuses over time. The main finding is that the risk premium may be underestimated or overestimated in the early stages of the model but as time increases the risk premium will converge to a steady value. The rate at which the risk premium converges depends on the speed at which information diffuses to entrepreneurs.

Key Words: Information Diffusion**,** Entrepreneurial opportunity, Risk Premium

1 Introduction

Contributions on the spread of information (rumours), in most cases, assume that agents assimilate new information through Bayesian updating. Banerjee (1993) purported that rumours spread through individual Bayesian updates and so used this mechanism to derive the underlying stochastic process driving the spread of the rumour. Other contributions such as Vettas (1998) explored a bilateral learning process where, agents on the consumption side learn (via Bayesian updating) by observing other agents on the consumption side, while agents on the production side learn by observing other agents on the production side. The dynamics of the rumour process was approximated by a differential equation. Another contribution, Kirman (1993), approached the spread of information through a recruitment mechanism.

Kirman (1993) purported that the dynamics of the spread of information can be determined by observing the behaviour of ants. Kirman examined a scenario where ants are faced with two symmetric food sources, the ant that discovers a food source interacts with other ant(s) enabling the information on the location of the food source to spread. Each ant that interacts with another ant that has the information is recruited with a certain probability. The number of ants that assimilate the information per unit time is the number of ants that gather around a particular food source per time. The differential equation governing the spread of the information was uncovered from these observations. Another form the recruitment mechanism took was that idealised by Ellison and Fudenberg (1995).

Under the heading, "Word-of-Mouth Communication and Social Learning" Ellison and Fudenberg (1995) formulated a model where, agents learn about their environment by communicating with other agents in the economy. Each agent observes payoffs based on their experiences and idiosyncratic shocks from consuming a particular good. As agents communicate, each agent will have information on the payoffs of a random sample of agents from the population per unit time. Each agent then finds the average of these payoffs and since the shocks are

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* Kario-Paul Brown is corresponding author. Email: kario-paul.brown@uwimona.edu.j

[1] Kosfeld, M. (2005) Rumours and Markets, *Journal of Mathematical Economics*, vol. 41, pp 646-664.

person specific the average shock is 0. Future decisions on whether to consume the good or service is based upon whether the average payoff is above a certain amount.

There are many studies modelling the spread of information in markets, but no study models the effects of the spread of information on the risk premium. Against this background this study will uncover the dynamic process that governs the behaviour of the risk premium over time as information diffuses. To my knowledge there is only one study that is closely related to this and it focuses on the effects of the spread of information on asset prices (Kosfeld 2005). All other contributions to the literature focused on the problem of market inefficiency as a by product of social learning.

Kosfeld (2005) formalised a model where information diffuses by word-of-mouth to an infinite number of agents overtime. Any agent infected with the rumour will spread the rumour to the agent immediately adjacent to her at an exogenously determined rate. The dynamics of the rumour process was captured through an equation similar to that of interacting particle systems (Liggett 1985). The effect on asset prices over time were analysed through the use of Arrow-Debreu securities.

The approach taken by this study as it relates to the rumour spreading dynamics is similar in sprit to that used by Kosfeld (2005) and Ellison and Fudenberg (1995). This study assumes that each entrepreneur gets the information that the other entrepreneur has through social interactions. Entrepreneurs in this model transmit qualitative messages which are interpreted as good or bad by the recipient of the messages. The receiving entrepreneur then uses this new information to calculate their probability beliefs over the states of nature.

The study is organized as follows. Section two provides the mathematical construct of the model. Section three outlines the fundamental equations of the solution. Section four provides a simulation of the model and the implications of the results. Section five gives a conclusion of the main results from the model and discusses possible extensions.

2 The Model

In a market with perfect information and uncertainty equilibrium is characterised by similar entrepreneurial opportunities having the same risk-adjusted returns. If both risky and risk-free[2] entrepreneurial opportunities are undertaken in equilibrium, then it means that there is a risk premium to account for the risk inherent in the risky entrepreneurial activity. Risk is essentially the volatility of unexpected outcomes and therefore captures possible movements in the risky entrepreneurial opportunity (probability distribution over different states of nature). When information is perfect entrepreneurs in the economy will be able the estimate the true risk inherent in the entrepreneurial opportunity. Since returns from the risky entrepreneurial opportunity are stationary then it means that each entrepreneur can observe past returns and use it to obtain the probability distribution of returns over the states of nature. The entrepreneur can use this probability distribution to calculate the volatility of this risky entrepreneurial opportunity and also the risk premium.

The preceding theoretical construct depends on two fundamental assumptions which are: (i) entrepreneurs observe returns from the risky entrepreneurial opportunity long enough to realise its stationarity; and (ii) information is perfect.

The volatility and the risk premium are determined from the probability distribution over the states of nature, which also depends on the information available to entrepreneurs. In an economy with imperfect information the probability that each entrepreneur associates with each state of nature might not reflect the true probability of that state occurring in reality. Therefore, one can treat the probabilities associated with each state of nature as beliefs of

[2] A risk-free entrepreneurial activity can be viewed as entering into a mature well developed market that has the capacity to absorb an additional player. In such a situation the average return to an entrepreneur is well known, hence the description as risk-free.

the likelihood of that state occurring in reality[3]. These probability beliefs are dependent upon the quality of information available to an entrepreneur at a particular time.

A Significant amount of investors gets information from other investors, therefore it reasonable to assume that entrepreneurs use information received from other investors in determining the probability distribution of wealth across each state of nature. In doing so entrepreneurs subject themselves to information imperfection, because the quality of information they have will depend on the number of persons they interact with. The more persons they interact with the better the quality of their information set and also the better their estimate of the probability distribution of wealth across the states of nature. Hence as time progresses and entrepreneurs interact the better their estimate of the probability distribution of wealth over the states of nature and the better their estimate of the risk premium.

This model will develop a framework where each entrepreneur possesses some qualitative information on a risky entrepreneurial opportunity not available to other entrepreneurs in the model. However, as entrepreneurs interact socially these pieces of qualitative information will spread to other entrepreneurs in the economy who will classify them as bad or good. Every entrepreneur then uses their information set to calculate their probability beliefs over the states of nature. In so doing the entrepreneur also calculates the volatility inherent in the risky entrepreneurial opportunity and also the desired risk premium. It is necessary to make some structural assumptions to facilitate model set-up. These assumptions are now presented.

Assumption 1: There are two entrepreneurial opportunities, $a = 0,1$ in the economy that give returns R_{sa} in each state of nature. Entrepreneurial opportunity $a = 0$ is the risk free entrepreneurial opportunity that gives a return of R_0 in all states of nature and entrepreneurial opportunity $a = 1$ is the risky entrepreneurial opportunity that gives returns R_{11} in state 1 and R_{21} in state 2. We will also assume that each entrepreneur has perfect information on the risk-free entrepreneurial opportunity but not on the risky entrepreneurial opportunity.

Assumption 2: Each identical entrepreneur, $1,......., P \in Z$, through social interaction, updates their probability beliefs over the states of nature $s = 1,2$. State 1 represents a good state and state 2 represents a bad state. People obtain qualitative information on the risky entrepreneurial opportunity and classify them as either good or bad then update their probability beliefs over the state of nature. The probability beliefs are given by equation (1) and (2). Equation (1) is the belief that nature is in a good state at time t where; $X(t)$ is the number of good pieces of information obtained, $N(t)$ is the number of entrepreneurs the average entrepreneur has interacted with either directly or indirectly. Equation (2) represents the belief that the economy is in a bad state.

$$p(t) = prob(good) = X(t)/N(t) \quad (1)$$

$$(1 - p(t)) = prob(bad) = 1 - \left[X(t)/N(t)\right] \quad (2)$$

$Info(t)$ is the set of all pieces of information the average entrepreneur possesses (determined from interaction whether directly or indirectly) from the initial period until time t, I_i is the piece of information that entrepreneur $i \in Z$ was endowed with, $\nu(t)$ is the set of all pieces of information that is classified as good at time t.

Assumption 3: There is a fixed set of information, Ω, in the universe. This information allows one to determine the probability distribution over the states of nature of the risky entrepreneurial opportunity. Nature will assign all pieces of information from this set randomly to the population before interaction begins. The distribution of all pieces of information is given by the set Θ. As entrepreneurs interact they will share knowledge on risky entrepreneurial opportunity. Good information will be assigned a value of 1 and bad information will be assigned a

[3] For a discussion on the existence of subjective probability, see Anscombe, F. & Aumann R. (1963) A Definition of Subjective Probability. *Annals of Mathematical Statistics* vol. 34 No. 1. pp 199-205

value of 0. If a significant amount of the entrepreneurs that a specific entrepreneur comes in contact with possess good information then the specific entrepreneur will believe with greater probability that nature is in a good state.

Hence: $Info(t) = \{ x \in \Theta \mid x_i = I_i \forall i = 1,...., N(t) \}$ (3)

$$\nu(t) = \{ x \mid x \in Info(t) \ \& \ x = 1 \} \quad (4)$$

$$X(t) = |\nu(t)| \quad (5)$$

Equation (3) represents the information set of the entrepreneurs who have interacted with each other up until time t. Equation (4) represents the set of all good information hence $\nu(t)$ is a subset of $Info(t)$. Equation (5) represents the amount of good information and therefore $X(t)$ is the cardinality of the set $\nu(t)$.

Assumption 4: Each entrepreneur will interact randomly with other entrepreneurs in the population and in so doing spread information to other entrepreneurs. Those entrepreneurs who get information through interaction also spread information to others through interaction and so information spreads randomly. P is the number of entrepreneurs in the population; each "knower" (N) interacts with k other entrepreneurs at a time where some of these k individuals may have the information already. Hence the number of informed entrepreneurs per period of time is given by (Nk). However, the proportion of uninformed individuals is equal to $(P-N)/P$, therefore the total amount of newly informed entrepreneurs is given by equation (6) where $\dot{N}(t)$ represents the derivative of the number of informed entrepreneurs with respect to time.

$$\dot{N}(t) = \frac{k}{P} N(P-N) \quad (6)$$

Assumption 5: Each entrepreneur is risk averse and possesses a Von Neman-Morgenstern utility function that characterizes the objective being maximized each period after forming probability beliefs over each state of nature. In addition, total wealth is exhausted on all entrepreneurial opportunities and the proportion of wealth allocated to each entrepreneurial opportunity is α_a, therefore each entrepreneur maximizes equation (7) (the expected utility conditional on the wealth and information available to entrepreneur i at time t) subject to equation (8).

$$\underset{\alpha_2}{Max}\ U(W_s) = E[u(W_s)] = p \cdot u(W_1) + (1-p) \cdot u(W_2) \quad (7)$$

St $W_s = w(0)[R_1 + \alpha_1 (R_{s1} - R_0)]$ (8)

Hence each entrepreneur solves the problem:

$$\underset{\alpha_1}{Max}\quad U(W_s) = p \cdot u\{w(0)[R_1 + \alpha_2 (R_{12} - R_1)]\} + (1-p) \cdot u\{w(0)[R_1 + \alpha_2 (R_{22} - R_1)]\} \quad (9)$$

The First order condition is:

$$p \cdot u\{W_s\}' [w(0)(R_{12} - R_1)] + (1-p) \cdot u\{W_s\}' [w(0)(R_{22} - R_1)] = 0 \quad (10)$$

This can be simplified to:

$$p \cdot u'(W_1) R_{11} + (1-p) \cdot u'(W_2) R_{21} = p \cdot u'(W_1) R_0 + (1-p) \cdot u'(W_2) R_0 \quad (11)$$

Solving for the risk free rate (equation 10):

$$R_0 = \frac{p \cdot u'(W_1) R_{11} + (1-p) \cdot u'(W_2) R_{12}}{p \cdot u'(W_1) + (1-p) \cdot u'(W_2)} \quad (12)$$

Since the risk premium is the expected return of the risky entrepreneurial opportunity minus the risk free rate. The risk premium is given by equation 13.

$$\text{Risk premium } RP = pR_{11} + (1-p)R_{12} - \frac{p \cdot u'(W_1)R_{11} + (1-p)u'(W_2)R_{12}}{p \cdot u'(W_1) + (1-p)u'(W_2)} \quad (13)$$

Equation (13) is essentially the one which determines the risk premium in an economy with only two (2) states of nature and two entrepreneurial opportunities (a risk-free entrepreneurial opportunity and a risky entrepreneurial opportunity). Modelling an economy with more than two states of nature and more than two entrepreneurial opportunities would be more complex. Since we are trying to capture the behaviour of the risk premium as information diffuses between entrepreneurs over time one only needs to be concerned with a risk free and risky entrepreneurial opportunity over two states of nature. The results derived from this simplified model remain unchanged when one allows for the complexities arising from more than two entrepreneurial opportunities and more than two states.

The risk premium will be affected by changes in the probability beliefs which ultimately depend on the number of persons that has interacted. As entrepreneurs interact with more individuals the probability beliefs will approach the true probability distribution over the states of nature. These probability beliefs are the same as those that would be observed from a stationary sequence over a long period of time. Hence, the risk premium will approach the true long run risk premium as entrepreneurs interact with other entrepreneurs over time.

3 The Fundamental Equations

The fundamental equations are the equations that govern the evolution of the risk premium over time. In order to find the fundamental equations one has to first find the solution for the differential equation which governs the evolution of the spread of information among entrepreneurs in the economy (equation (6)). When equation (6) is solved for the number of informed entrepreneurs per time period we get equation (14)[4] where 'b' and 'c' are constants that can be determined from initial conditions.

$$N(t) = \frac{P}{1+be^{-ct}} \quad (14)$$

Hence equation (1) can be rearranged to equation (15)

$$p(t) = \frac{\left[1+be^{-ct}\right]X(t)}{P} \quad (15)$$

This also means that equation (13) can be modified to equation (F1) and hence the fundamental equations are given by (F1) to (F4):

$$RP = \left(\frac{1+be^{-ct}}{P}\right)X(t)R_{11} + \left(\frac{P-1-b^{-ct}}{P}\right)X(t)R_{12} - \frac{\left(\frac{1+be^{-ct}}{P}\right)X(t)\cdot u'(W_1)R_{11} + \left(\frac{P-1-be^{-ct}}{P}\right)X(t)u'(W_2)R_{12}}{\left(\frac{1+be^{-ct}}{P}\right)X(t)\cdot u'(W_1) + \left(\frac{P-1-be^{-ct}}{P}\right)X(t)u'(W_2)} \quad \text{(F1)}$$

4 See appendix for derivation

Where $X(t)$ is governed by:

$$Info(t) = \{ x \in \Theta \mid x_i = I_i \forall i = 1,...., N(t) \} \quad \text{(F2)}$$

$$\nu(t) = \{ x \mid x \in Info(t) \; \& \; x = 1 \} \quad \text{(F3)}$$

$$X(t) = |\nu(t)| \quad \text{(F4)}$$

4 Simulation of the Model

The model was simulated under some additional assumptions. First it was assumed that there was 10,000 investors in the market each having a utility function given by $u(w) = \log(w)$ and equal wealth. This utility function is used because each investor is risk averse and this utility function exhibits constant absolute risk aversion. The average daily return to stock prices were used for the risky entrepreneurial opportunity and the average daily change in the price of zero coupon bonds government bonds was used for the risk free entrepreneurial opportunity. The daily returns in stock prices were assumed to be 3% in a good state and 1.5% in a bad state and the daily change in the price of risk free entrepreneurial opportunity was assumed to be %0.5 in all states. A computer simulation is used to determine the distribution of information among the population and also to simulate the model.

Results

When the distribution of information is normally distributed among the population meaning that there are equal amount of good and bad information we get the results in figure 1 and table 1.

Table 1:

Time Period(s)	Risk Premium in Basis Points
0	341
1	397
2	332
3	346
4	344
5	343
6	342
7	342
8	342

Table 1 shows one possible movement in the risk premium over an 8 period horizon as investors adjust to the information that other entrepreneurs have. The table shows that the risk premium will stabilise just after the 5^{th} period has elapsed. Figure 1 provides a graphical analysis of the risk premium over the 8 period horizon as entrepreneurs adjust to the information. Figure 1 shows that the risk premium might be overestimated in the early period, however, as individuals adjust to the information there will be some small variations in the value of the risk premium up until the 5^{th} period. It is important to note that graph and table show only one possible movement in the value of the risk premium. If the information set changes then the trajectory of the information set changes. Multiple runs of the simulation under different normally distributed information sets give different trajectories but majority of trajectories will stabilise by the 6th period; see Table 2.

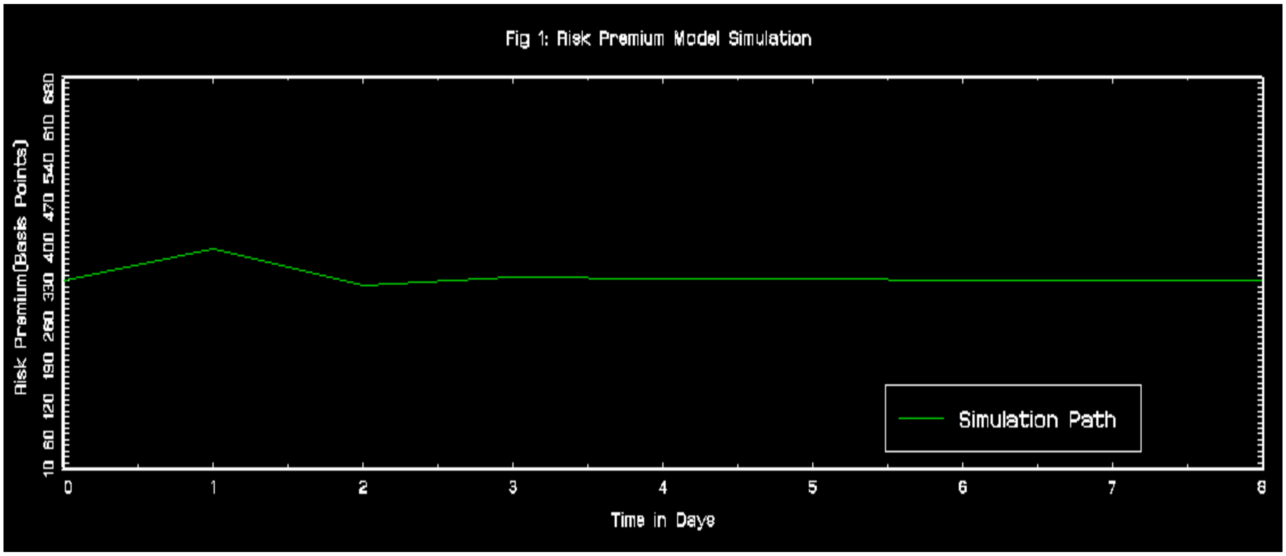


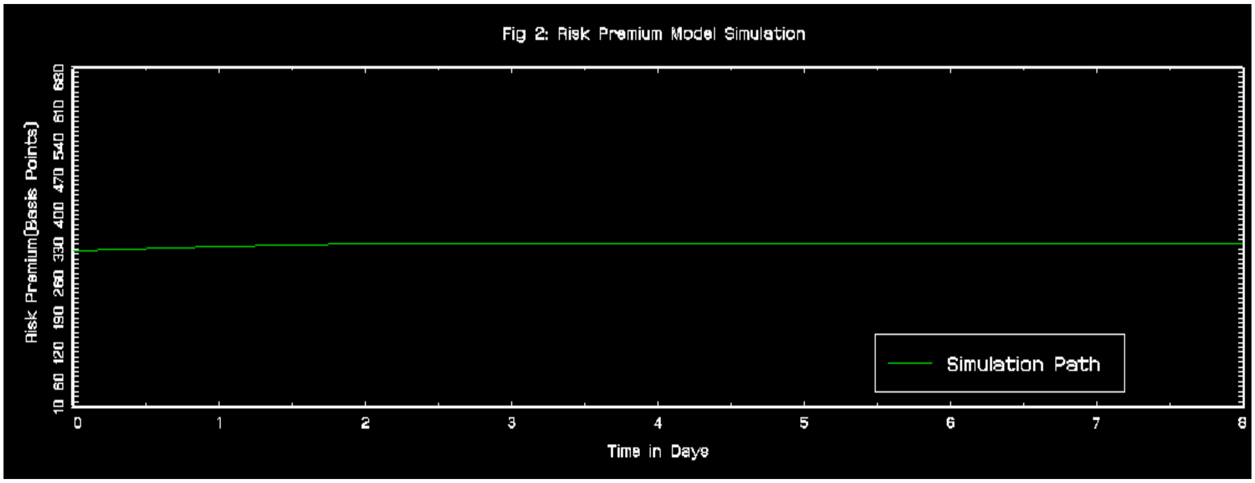
Table 2: Risk Premium for 10 different simulations

Time	R P1	RP2	RP3	RP4	RP5	RP6	RP7	RP8	RP9	RP10
0	341	500	150	150	281	281	397	281	397	344
1	397	383	250	296	341	383	341	250	424	397
2	332	394	296	346	330	313	313	313	378	341
3	346	357	301	369	308	339	351	345	381	279
4	344	320	318	357	320	341	343	339	373	339
5	343	326	327	358	334	349	328	335	361	359
6	342	338	336	350	339	345	334	335	347	343
7	342	343	339	344	337	341	335	339	344	343
8	342	342	339	341	337	343	336	340	341	343

The preceding analysis was based on the initial condition that only 3 persons interacted in the initial period and by the end of the first day 8 persons have interacted with each other. To extend the analysis to larger more developed entrepreneurial opportunities we will assume that there are 100,000 investors facing the same set of assumptions. In the initial period 500 investors have interacted with each other and at the end of the first day 1000 investors have interacted with each other. One possible trajectory revealed that there relatively no changes in the risk premium implying that entrepreneurs adjust to information almost instantly (see table 3 and figure 2).

Table 3:

Time Period(s)	Risk Premium in Basis Points
0	328
1	336
2	341
3	342
4	340
5	340
6	340
7	340
8	340

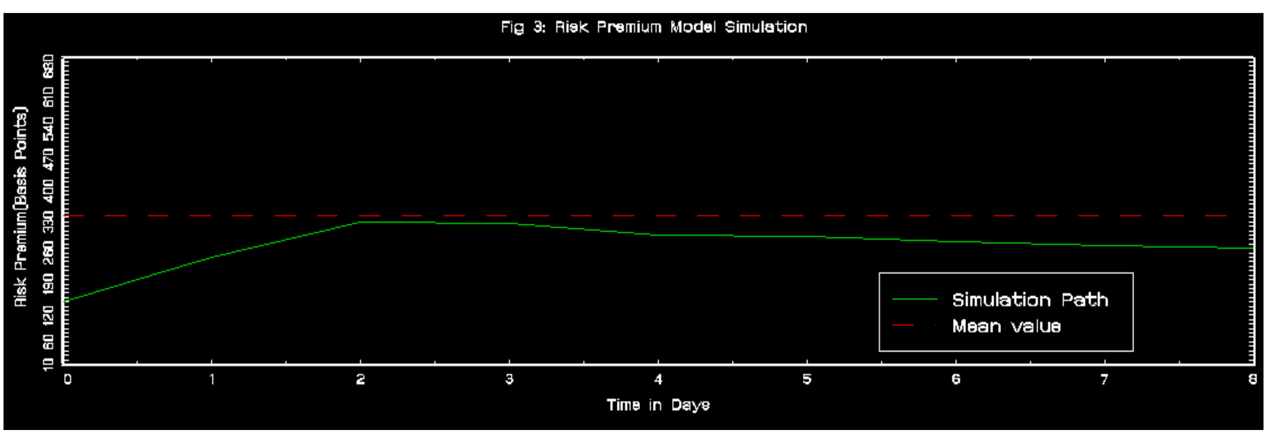


If many different simulations were conducted using different information sets then one would notice that information spreads almost instantaneously in almost all cases. This is exemplified through the relatively stable risk premium over time (see table 4).

Table 4: Risk Premium for 10 different simulations

Time	R P1	RP2	RP3	RP4	RP5	RP6	RP7	RP8	RP9	RP10
0	328	346	353	335	329	339	358	341	353	348
1	336	349	350	338	336	339	349	338	349	348
2	341	346	342	337	336	341	343	346	352	346
3	342	347	339	340	338	342	342	344	344	340
4	340	344	339	339	340	341	341	343	340	341
5	340	342	340	340	339	342	341	342	339	341
6	340	341	341	341	340	341	341	341	340	341
7	340	342	341	340	340	341	341	341	340	341
8	340	341	341	341	340	341	341	341	340	341

Another finding from the simulations is that the risk premium is highest when the probability belief is derived from a normally distributed information set. This is so because investors become more uncertain as to what state of nature will occur. However, when the information set is skewed whether positive or negative the risk premium would be substantially lower. This is so because the investor becomes more certain that a bad or a good state will occur. This fact can be seen from figure 3. The dotted line is the average risk premium when information is distributed symmetrically while the solid line is the distribution of the risk premium when information is skewed to the left or to the right.



5 Conclusion

The behaviour of the risk premium depends upon the distribution of information before interactions take place. In the event that the distribution of information chosen by nature is positively or negatively skewed, the risk premium would behave erratically in the early stages. However, as time increases the risk premium will converge to a steady level. The steady value of the risk premium would be substantially less than that obtained when the distribution of information is symmetric. On the other hand, if the distribution of information is symmetric then the risk premium would behave erratically in the early stages. As time progresses though, the risk premium would converge to a steady value, one that is greater than that when the distribution of information is skewed.

This means that if there is new information on a particular entrepreneurial opportunity and this new information is viewed as bad or good by majority of the market, then each entrepreneur will become more confident that a bad or good state will occur and so will demand less risk premium for holding undertaking the activity. On the other hand, if this new information is viewed as bad or good equally among the population then investors will become more uncertain about a particular state and so demand more risk premium for undertaking the entrepreneurial activity.

Another fundamental result is that in a large market information spreads rapidly and hence adjustments to information take place almost instantaneously. This is in sharp contrast to a small entrepreneurial opportunity market where information spreads less rapidly and so adjustments to information are much slower. This result implies that markets for entrepreneurial opportunities of developing countries adjust slower and so arbitrage opportunities will be eliminated at a slower pace than in developed countries.
As for possible extensions, it would be interesting to examine what would happen to the risk premium as entrepreneurs allocate wealth over time. It would also be interesting to see what effect the subjective time preference would have on the risk premium if any at all. In addition, the model could be extended by allowing the entrepreneurs to forget some of the information diffused through interaction. The effects of different forget rates on the risk premium could then be determined.

References

Anscombe, F. & Aumann R. (1963) A Definition of Subjective Probability. *Annals of Mathematical Statistics* vol. 34 No. 1. pp 199- 205

Black, F. (1972) Capital market equilibrium with restricted borrowing, *Journal of Business* vol. 45, pp 444-454.

Banerjee, V. (1992) A simple model of Herd Behaviour, *The Quarterly Journal of Economics*, issue 3.

Banerjee, V. (1993) The Economics of Rumours, The Review of Economic Studies, vol.60, No. 2. pp 309-327.

Brennan, M. (1971) Capital market equilibrium with Diverged Borrowing and Lending rates, *Journal of Financial and Quantitative Analysis*, pp 1197-1205.

Herstein, I. & Milnor J. (1953) An Axiomatic Approach to Measurable Utility, *Econometrica* vol. 21, No. 2. pp 291-297.

Jorion, P. (2007) Value at Risk. 3rd Edition: McGraw-Hill International Edition.

Karni, E. (1993) A Definition of Subjective Probabilities with State Dependent Preferences, *Econometrica*, vol. 61, pp 187-198.

Kirman, A. (1993) Ants, Rationality and Recruitment, *The Quarterly Journal of Economics*, vol.108, No. 1. pp 137-156.

Kofeld, M. (2005) Rumours and Markets, *Journal of Mathematical Economics*, vol. 41, pp 646-664.

Mishkin, F. (2004) The Economics of Money Banking and Financial Markets. 7th Edition: Addision Wesley Series.

Sharpe, W. (1964) Capital entrepreneurial opportunity prices: a theory of capital market equilibrium under conditions of risk, *The Journal of Finance* vol. 19, pp 425-442.

Varian, H. (1987) The Arbitrage Principle in Financial Economics. *Journal of Economic Perspectives*, 1(2): 55–72.

Vettas, N. (1998) Demand and Supply in New Markets: Diffusion with Bilateral Learning *The RAND Journal Of Economics*, vol. 29, No. 1. pp 215-233.

6 Appendix

A1: Proof that the solution to $\dot{N}(t)=\frac{k}{P}N(P-N)$ is $N(t)=\frac{P}{1+be^{-ct}}$

$$\frac{dN}{dt}=\frac{k}{P}N(P-N)$$ Equation (1)

Since $\frac{k}{P}$ is constant then let $K=\frac{k}{P}$

Hence equation (1) becomes $\frac{dN}{dt}=KN(P-N)$ Equation (2)

Rearranging equation (2) we get $\frac{1}{N(P-N)}dN=Kdt$ Equation (3)

Integrating equation (3) we get $\int\frac{1}{N(P-N)}dN=\int Kdt$ Equation (4)

This gives $\frac{1}{P}\cdot\ln\left(\frac{N}{P-N}\right)=Kt+C \Leftrightarrow \ln\left(\frac{N}{P-N}\right)=PKt+PC$ Equation (5)

Taking exponents we get $\frac{N}{P-N}=e^{PKt}\cdot e^{PC}$ Equation (6)

Since is e^{PC} a constant we have $\frac{N}{P-N}=Ae^{PKt}$ Equation (7)

Solving for N $N=(P-N)Ae^{PKt} \Leftrightarrow N(1+Ae^{PKt})=PAe^{PKt}$ Equation (8)

$$N=\frac{PAe^{PKt}}{1+Ae^{PKt}} \Leftrightarrow N=\frac{P}{1+\frac{1}{A}e^{-PKt}}$$ Equation (9)

Let $PK=c$ and $1/A=b$ then we get equation (14)