# Market Makers as Entrepreneurs in Financial Markets

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**Abstract**

This paper investigates the choice that market makers take when deciding what assets to trade in the face of the diversity in the risk profile of investors. Specifically, financial assets are dichotomized into relatively high risk and relatively low risk assets. Consequently, a model of vertical product differentiation in the tradition of Bresnahan (1987) is deployed in the analysis. The main finding of the paper is that Market Makers will optimally choose to specialize in different financial assets. That is, in equilibrium some market makers, although possessing a portfolio that spans the full range of assets, prefer to facilitate trades in assets which exhibit low volatility such as government paper and triple "A" rated paper. On the other hand, some market makers prefer to specialize in assets that constitute far more significant risks. These conclusions speak directly to managers of pension funds not being allowed to take on the riskiest of assets and hedge fund managers accepting far greater risks. The paper also shows that this dichotomous choice on the part of market makers allows for both sets of market makers to generate greater profits.

*JEL Classifications: G10, G11, G12.*
*Key Words: Market Maker, Investor Preferences, Bid-Ask Spread, Risk, Asset Prices.*

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# 1 Introduction and Literature Review

This paper investigates the choice that market makers take when deciding what assets to trade in the face of the diversity in the risk profile of investors. Market Makers have tended to specialize in different financial assets for various reasons. Some market makers, although possessing a portfolio that spans the full range of assets, prefer to facilitate trades in assets which exhibit low volatility such as government paper and triple "A" rated paper. On the other hand, some market makers prefer to specialize in assets that constitute far more significant risks.

Similarly, investors have different preferences, or more specifically, they have different appetites for risks. Investors' risk appetites influence the assets they choose to hold in their respective portfolios. As a case in point, pension funds are not allowed to take on the riskiest of assets and hedge funds that cater to sophisticated investors might allow for investment in assets that constitute far greater risks. A great deal of literature has focused on the demand for assets with varying degrees of risks. These assets, generally speaking, can be dichotomized into high risk and low risk assets where the level of risk is normally measured using the volatility in returns. In the category of risky assets one may identify stocks and mutual funds while in the category of low risk one may look at Treasury Securities. As such, risky assets are dichotomized into two types and analyzed on this basis.

Chen and Lu (2004) looks closely at wealth composition and the demand for risky assets. Their analysis is premised on a two-period analytical framework which deliberately explores the relationship between an individual's wealth composition and portfolio selection. In this context, findings purport that the wealth composition of an individual has tremendous influence on one's optimal portfolio choice. In view of this, Chen and Lu (2004) argue that an individual with a high ratio of human wealth to financial wealth should increase the proportion of risky asset holdings. These results are contradicted by Bodie et. al (1992) which asserts that wealth composition is closely related to portfolio choice, but is independent of demand for risky assets. Together, the Bodie et. al. (1992) finding and the Chen and Lu (2004) results help to set the foundation for our research in relation to market makers and their asset choice-contingent on investor risk profiles.

Jagannathan and Kocherlakota (1996) uses standard economic models of investor behaviour to show the type of assets, whether risky or relatively riskless, that investors will choose. In this model age and wage distribution become very important in investor choice. The findings suggest that as a person ages there will be a shift in the risk composition of financial wealth in order to offset the decline in the value of human capital. The methodology employed in this paper was first derived by Merton (1971). The implications of this research is that younger persons will tend to invest in more risky assets as a result of the labour years ahead of them that can offset losses. However, as an individual ages less risky assets will be chosen due to the depletion of human capital and labour income flow.

Chen and Lu (2004) and Jagannathan and Kocherlakota (1996) imply that market makers will choose to hold high risk and low risk assets as a result of the fact that the securities market is essentially made up of different types of investors - those who will invest in risky assets and those that will invest in less risky assets. In this event, and for the purpose of this research, the market for investment assets can be characterized as a duopoly situation where investors are uniformly spaced along a line segment. Investors will then interact with market makers and/or brokers who situate themselves as either specialists in high risk or low risk securities.

If market makers situate at the same position across the distribution of investors then it is anticipated that each will have greater market share, but that there will be unbridled competition between them. Alternatively, if market makers situate themselves at different points across the space which represents investor preferences, each may have lower market share, but greater ability to influence market outcomes. It is significant to note that the idea that asset prices are unpredictable, or that the efficient markets hypothesis holds, renders the usual ability to influence price as articulated in models of product differentiation questionable in the context of financial markets.

A large volume of literature has been written on product differentiation and its associated tools. Notable is Hotelling (1929), who speaks about a fixed number of firms competing on a finite linear market by location alone. Specifically, Hotelling (1929) analyzes the behaviour of duopolists locating a single store on a finite one-dimensional geographic market. The main finding of this paper was that firms within an industry would seek to distinguish themselves minimally. A major limitation to the Hotelling (1929) model as found in Prescott and Visscher (1977)was in relation to the location assumption which makes the model inappropriate for much actual market product differentiation.

Prescott and Visscher (1977) presents a variant of the Hotelling (1929) model. In their paper the authors modified Hotelling's structure in order to ensure continuity. Prescott and Visscher (1977)use an equilibrium model of firms in which each firm locates in sequence with correct expectations of the way its decisions influence the decisions of firms yet to locate. That is, firms in the model locate one at a time and relocation is assumed to be 'prohibitively' expensive. After all, the findings of this model suggest that duopolists seek maximum separation instead of the minimum separation claimed by Hotelling.

Thomas (2009a), highlights that Hotelling's model was widely criticized by D'Aspremont, Gabszewics and Thisse (1979), hereafter called DGT (1979) . Particularly, Thomas states that Hotelling's model was widely viewed as one that speaks to horizontal product differentiation. However, DGT (1979) and Salop (1979) introduced what is today widely known as vertical product differentiation. The technique employed by Bresnahan (1987) is one that deals fundamentally with vertical product differentiation. In this paper the aim was to analyze competition and collusion in the American automobile industry. Specifically, the paper provides a model of non-price taking supply of differentiated products assuming cooperative behavior and shows how the hypothesis of competition can be empirically separated from that of collusion. However, a major limitation of Bresnahan's model as outlined in Thomas (2009a) was in terms of the model not being applicable to industries where there exists multi-dimensional product differentiation.

The current research can be viewed as bringing the techniques deployed in models of product differentiation to bear on financial markets. The idea is that investors have preferences spread across risk space and market makers then strategically position themselves to take advantage of investor risk preferences in order to exploit profit opportunities. The main conclusion of this paper is that market makers will maximally differentiate themselves across risk space. It is also shown that it would not be wise for the low risk market maker to try to imitate the high risk market maker by trying to trade high risk assets. That is, the low risk market maker is less efficient at trading high risk assets than the high risk market maker.

Furthermore, the outcome of maximal product differentiation speaks to a dampening of competition. This result holds up as the compensation that can be received by maximally differentiating far outweighs that which can be achieved under fierce competition. The remainder of the paper is organized as follows: Section 2 presents the model and explores the equilibrium outcomes; Section 3 investigates how those outcomes are affected by changes in risk preferences via Comparative Statics analysis; and Section 4 concludes.

## 2 The Model

A standard model of vertical product differentiation[1] is one in which investors choose exactly one product from a single firm at any one point in time and investors are uniformly distributed along a line segment. The investor who is most risk averse will be located at one end while the investors who is least risk averse will be located at the other end of the line segment. Therefore, if investors are forced to trade an asset that does not exactly match their risk appetite they will experience some disutility. Consequently, the market makers will want to distribute themselves along the line segment such that their strategic position in the market allows them to extract optimal benefits. Thus, the market makers will vertically differentiate their products in line with the investors preferences (Prescott and Vischer, 1977).

Bresnahan (1987) applied the model of Prescott and Vischer (1977) when he was analyzing competition and collusion in the American automobile

[1] See, Bresnahan (1987), inter alia.

industry. Subsequent alterations have been made to the model of Bresnahan (1987) by quite a number of authors. The model that will be used here is very similar to the model by Thomas (2009b).

In what follows we present a two stage game in which firms choose their type in the first stage and choose prices in the second stage. As will be discussed in later sections, in stage 2 types are exogenous variables which will affect prices as in the standard models. However, because we posit that marginal costs are not only different, but that they are functions of type; marginal costs will also be affected by shifts in these type. To formalize the model consider a two stage game where firms, $j \epsilon J = \{L, H\}$, (where 'L' represents the firm who takes on a portfolio consisting of relatively riskless assets and 'H' represents the firm who takes on a portfolio consisting of risky assets) make the following choices:

**Stage 1**: Each market maker $j$ chooses type, $t_j$ such that $t_j \, \epsilon \, [L\,, H\,] \subset \Re_+$, and denote the other market maker's type by $t_{-j}$.[2]

**Stage 2**:
Having chosen its type, and also observing the type of the other market makers, each market maker $j$ chooses its price[3], $O_i \, \epsilon \, \Re_+$, for each asset, given its own type and that of the other market maker.

The market maker buys and sells a set of assets, $A_i$, at the respective bid offer rates and immediately generates a return equal to the quantity times the bid-offer spread. However, there is a chance that the asset value will go up or down triggering losses or gains to the market maker. As such, the market maker maximizes:

$$\Pi_{LorH} = \sum_{i=1}^{n} (\rho_i) u_i O_i A_i + \sum_{i=1}^{n} (1 - \rho_i) d_i O_i A_i - \sum_{i=1}^{n} B_i A_i$$

Where[4]: $\rho_i$ is defined as the probability that the offer price will appreciate

$u_i$ is defined as the gross rate of appreciation of the offer price

$d_i$ is defined as the gross rate of depreciation of the offer price

$O_i$ is defined as the offer price as at the time the particular asset was purchased

[2]Assume that each market maker chooses to trade a particular type of asset because it has all the information about that particular type of asset. If it chooses to trade another type of asset it will have asymmetric information about the asset. This can cause the market maker to be less efficient at trading the other types of assets. That is, lack of information about other types of assets can cause the market maker to price these assets incorrectly. This may cause arbitrage opportunities to arise and put the market maker at a clear disadvantage. Consequently, the market maker chooses a particular set of assets which then determines the market maker's type.

[3]It is assumed that bid and ask prices are not chosen independent of each other. Therefore, only one of these prices is chosen and the other is simply determined by the bid-ask spread.

[4]It should be noted that the definitions below are stated within the confines of what a specific market maker does so that it is not necessary to introduce a second subscript to denote each market maker.

$B_i$ is defined as the bid
$A_i$ is defined as the financial assets that are traded.

Being that price is the strategic variable in the Bertrand Strategic Game, the typical market maker within a particular category will want to maximize its expected profit function with respect to price of the assets it has within its portfolio. In this way the firm will be able to see the variables that influence the price that it would want to sell an asset for. The variables that affect the price that the firm will sell an asset for, are the ones that it will use to determine what type of assets it should trade. The concept of Sub-game Perfect Equilibrium will be used to solve the model. Thus, we will solve for prices first and then, we will solve for the type of firm in order to arrive at an equilibrium. Comparative statics will be done in order to see how changes in these variables affect the price that will be chosen. Prices, however, depend on demands and therefore we turn to a discussion of investors and their demands for risky assets.

Each investor will receive payoffs as follows:

$$\begin{cases} \alpha_i \ \hat{R}_i \ H_i \text{ - } O_{H_i} & \text{if a high risk asset is chosen} \\ \alpha_i \ \hat{R}_i \ L_i \text{ - } O_{L_i} & \text{if a low risk asset is chosen} \end{cases}$$

Where: $\hat{R}_i$ is a measure of the risk preference of the investor, it is uniformly distributed on the unit interval. That is, $\hat{R}_i \sim$ r(0,1) $\alpha_i$ is investor heterogeneity parameter; $\alpha_i > 0$ $H_i$ is a measure of the benefit derived from trading in a high risk asset. $L_i$ is a measure of the benefit derived from trading in a low risk asset.

We seek to determine the indifferent investor so as to set up the demand functions for each type. This is achieved by obtaining an expression for the risk profile of the risk-neutral investor (indifferent between choosing high and low risk asset). That is, equating the payoffs gives:

$$\alpha_i \hat{R}_i H_i - O_{H_i} = \alpha_i \hat{R}_i L_i - O_{L_i}$$

Solving for $\hat{R}_i$ we obtain:

$$\hat{R}_i = \frac{O_{H_i} - O_{L_i}}{\alpha(H_i - L_i)}$$

The demand for high risk investors is given by:

$$A_{H_i}(O_{H_i}, O_{L_i}) = \begin{cases} 1 & \text{if } R_i \geq 1 \\ 1\text{-}R_i & \text{if } 0 < R_i < 1 \\ 0 & \text{if } R_i \leq 0 \end{cases}$$

The demand for low risk investors is given by:

$$A_{L_i}(O_{H_i}, O_{L_i}) = \begin{cases} 0 & \text{if } R_i \geq 1 \\ R_i & \text{if } 0 < R_i < 1 \\ 1 & \text{if } R_i \leq 0 \end{cases}$$

We now explicitly distinguish the profit functions of the two types of market-makers, one type dealing in high risk assets and the other type dealing in low risk assets. The profit that each market maker makes on the trade of a portfolio is given by:

$$\Pi_{H_i}(O_{H_i}, O_{L_i}) = \sum_{i=1}^{n} \rho_i u_i O_{H_i} A_{H_i} + \sum_{i=1}^{n} (1-\rho_i) d_i O_{H_i} A_{H_i} - \sum_{i=1}^{n} B_{H_i} A_{H_i}$$

$$\Pi_{L_i}(O_{H_i}, O_{L_i}) = \sum_{i=1}^{n} \rho_i u_i O_{L_i} A_{L_i} + \sum_{i=1}^{n} (1-\rho_i) d_i O_{L_i} A_{L_i} - \sum_{i=1}^{n} B_{L_i} A_{L_i}$$

The next step is to substitute demands into the profit functions and to use the First Order Conditions (F.O.C.) to solve for the offer prices of each type. The profit function for the market maker that deals in high risk assets after substitution then becomes:

$$\Pi_{H_i} = \sum_{i=1}^{n} \left\{ [\rho_i u_i + (1-\rho_i) d_i] O_{H_i} - B_{H_i} \right\} \left\{ 1 - \frac{O_{H_i} - O_{L_i}}{\alpha(H_i - L_i)} \right\}$$

The F.O.C. is given by[5]:

$$\frac{\partial \pi_H}{\partial O_{H_i}} = \left\{ \rho u + (1-\rho) d \right\} \left\{ 1 - \frac{O_H - O_L}{\alpha(H-L)} \right\} + \left\{ [\rho u + (1-\rho) d] O_H - B_H \right\} \left\{ \frac{-1}{\alpha(H-L)} \right\} = 0$$

$$\Leftrightarrow [\rho u + (1-\rho) d][\alpha(H-L) - O_H + O_L] = [\rho u + (1-\rho) d] O_H - B_H$$

$$\Leftrightarrow [\rho u + (1-\rho) d][\alpha(H-L) + O_L] + B_H = 2[\rho u + (1-\rho) d] O_H$$

$$O_H = \left\{ \frac{\alpha(H-L) + O_L}{2} \right\} + \left\{ \frac{B_H}{2[\rho u + (1-\rho) d]} \right\}$$

The profit function for the market maker that trades low risk assets then becomes:

[5] To simplify the analysis it will be assumed that each market maker invests in only one asset, and therefore we no longer need to use the subscript $i$ to distinguish between the various assets. This can also be interpreted such that there is a representative asset, thus there is equal probability of price movements in all directions for all assets. Another interpretation is that asset prices are determined independently of each other and consequently a representative asset "i" is characterized as appears, and we drop the subscript "i" for ease of notation.

$$\Pi_{L_i} = \sum_{i=1}^{n} \left\{ [\rho_i u_i + (1-\rho_i)d_i]O_{L_i} - B_{L_i} \right\} \left\{ \frac{O_{H_i} - O_{L_i}}{\alpha(H_i - L_i)} \right\}$$

The F.O.C. is given by:

$$\frac{\partial \pi_L}{\partial O_{L_i}} = \left\{ \rho u + (1-\rho)d \right\} \left\{ \frac{O_H - O_L}{\alpha(H-L)} \right\} + \left\{ [\rho u + (1-\rho)d]O_L - B_L \right\} \left\{ \frac{-1}{\alpha(H-L)} \right\} = 0$$

$$\Leftrightarrow [\rho u + (1-\rho)d][O_H - O_L] = [\rho u + (1-\rho)d]O_L - B_L$$

$$\Leftrightarrow 2[\rho u + (1-\rho)d]O_L = B_L + [\rho u + (1-\rho)d]O_H$$

$$O_L = \left\{ \frac{B_L}{2[\rho u + (1-\rho)d]} \right\} + \left\{ \frac{O_H}{2} \right\}$$

Let the expected price change for an asset be given by K. That is: K $\equiv \rho$ u + (1 - $\rho$)d

$$O_L = \left\{ \frac{B_L}{2K} \right\} + \left\{ \frac{\alpha(H-L) + O_L}{4} \right\} + \left\{ \frac{B_H}{4K} \right\}$$

$$\Leftrightarrow \frac{3}{4}O_L = \left\{ \frac{2B_L + B_H}{4K} \right\} + \left\{ \frac{\alpha(H-L)}{4} \right\}$$

$$\Leftrightarrow O_L = \left\{ \frac{2B_L + B_H}{3K} \right\} + \left\{ \frac{\alpha(H-L)}{3} \right\}$$

$$O_H = \left\{ \frac{\alpha(H-L)}{2} \right\} + \left\{ \frac{2B_L + B_H}{6K} \right\} + \left\{ \frac{\alpha(H-L)}{6} \right\} + \left\{ \frac{B_H}{2K} \right\}$$

$$O_H = \left\{ \frac{2\alpha(H-L)}{3} \right\} + \left\{ \frac{B_L + 2B_H}{3K} \right\}$$

Thus, the low risk market makers' asset price is given by:

$$O_L = \left\{ \frac{\alpha(H-L)}{3} \right\} + \left\{ \frac{2B_L + B_H}{3K} \right\}$$

And, the high risk market makers' asset price is given by:

$$O_H = \left\{ \frac{2\alpha(H-L)}{3} \right\} + \left\{ \frac{B_L + 2B_H}{3K} \right\}$$

It can be shown that when $\rho$ the probability of an appreciation is a specific form, which ensures that it is non-zero (and, of course positive) then $K \geq 1$. This implies that the expected ask price of the market makers is positive. Otherwise, they will not trade that particular asset. That is, market makers would not transact an asset for which there is no chance that the price will appreciate in value. To see this suppose:

$$\rho \geq \frac{1-d}{u-d}$$

$$\Leftrightarrow \rho u + (1-\rho)d \geq 1; \Leftrightarrow K \geq 1.$$

**Assumption 1**: $\alpha K/2 < B_L' < 2\alpha K$

This assumption says that if the risk contained in the low risk asset, as denoted by $L$, is increased then the bid price on that asset must increase. Specifically, by an amount in proportion to (greater than $\alpha K/2$) how the expected price change (K) interacts with the heterogeneity of investors ($\alpha$). The assumption also ensures that such a price change is capped in relation to this same quantity ($< 2\alpha K$). This in a sense says that there are no wild swings in asset prices as risks change. That is, small changes in risks will lead to small changes in asset prices.

**Proposition 1**: *Under Assumption 1, as one market maker trades more of the types of assets consistent with its own risk profile, the other type of market maker will be able to charge a higher ask price.*

*Proof of proposition 1*:

For the low risk market maker

$$\frac{\partial O_L}{\partial H} = \frac{\alpha}{3} + \frac{B_H'}{3K} = \frac{\alpha K + B_H'}{3K} \geq 0$$

Thus, when the high risk market maker trades in riskier assets, the low risk market maker can charge a higher price.

For the high risk market maker

$$\frac{\partial O_H}{\partial L} = \frac{-2\alpha}{3} + \frac{B_L'}{3K} = \frac{-2\alpha K + B_L'}{3K} \leq 0$$

If the low risk market maker tries to trade in riskier assets then the high risk market maker is forced to charge a lower price.

**Proposition 2**: *The ask prices allowed by market makers increases with its own bid prices as well as with the other market makers' bid prices.*

*Proof of proposition 2*:

$$\frac{\partial O_L}{\partial B_L} = \frac{2}{3K} > 0$$
$$\frac{\partial O_L}{\partial B_H} = \frac{1}{3K} > 0$$
$$\frac{\partial O_H}{\partial B_H} = \frac{2}{3K} > 0$$
$$\frac{\partial O_H}{\partial B_L} = \frac{1}{3K} > 0$$

**Proposition 3**: *Ask prices are increasing in investors' heterogeneity.*

*Proof of proposition 3*:

$$\frac{\partial O_H}{\partial \alpha} = \frac{2(H-L)}{3} > 0$$
$$\frac{\partial O_L}{\partial \alpha} = \frac{(H-L)}{3} > 0$$

**NB**: High risk market makers ask prices increases at a faster rate than low risk firms as heterogeneity increases among investors.

**Proposition 4**: *Under Assumption 1, the higher the ask price faced be market makers in trading a particular asset, the higher the ask price the market maker will charge.*

*Proof of proposition 4*:

$$\frac{\partial O_L}{\partial L} = \frac{-\alpha}{3} + \frac{2B_L'}{3K} = \frac{-\alpha K + 2B_L'}{3K} > 0$$
$$\frac{\partial O_H}{\partial H} = \frac{2\alpha K + 2B_H'}{3K} > 0$$

**Proposition 5**: *As the gap between the risk types widens, market makers can charge a higher price.*

*Proof of proposition 5*:

$$\frac{\partial O_L}{\partial (H-L)} = \frac{\alpha}{3}$$
$$\frac{\partial O_H}{\partial (H-L)} = \frac{2\alpha}{3}$$

# 3 Comparative Statics

It is instructive to use comparative statics to investigate the impact of changes in risk choices by market makers on the outcomes arrived at in the previous section. One should note that in the second stage (price choice stage) of the game risk profile choices by market makers are exogenous and therefore comparative statics will guide the solutions to the choice problem. In order to generate clear results and understand the parameter restrictions which drive the outcomes in this respect we proceed with the following assumption:

**Assumption 2**:

$$max\left\{\frac{2B'_L-\alpha K}{3\alpha K},\frac{3(H-L)B'_H}{2\alpha K(H-L)+B_L-B_H}\right\} < R < \frac{2\alpha K-B'_L}{3\alpha K} < \frac{2\alpha K+B'_H}{3\alpha K}$$

Assumption 2 in its simplest form says that both the high risk and low risk market maker faces positive demand. This is so as Assumption 1 will result in Assumption 2 implying that $0 < R < 1$. That is, from Assumption 1 (without considering the denominator) the first term in max{}, $2B'_L - \alpha K$, as appears in Assumption 2 is $> 0$, and therefore $R > 0$. On the other hand, on the right hand side of $R$ in Assumption 2, since by Assumption 1, $2\alpha K > B'_L$, then $R < 1$. Thus, we can conclude that Assumption 2 simply says that in equilibrium both types of market makers are supported.

**Theorem 1**: *Market Makers will choose to maximally differentiate themselves.*

This theorem will be explored and proven by investigating the following two propositions:

**Proposition 6**: *The two market makers will choose different types, t ϵ (L, H), which is a unique sub-game perfect Nash equilibrium.*

**Proposition 7**: *The greater the distance between risk types of the market makers, the greater will be the profitability of both market makers.*

*Proof of proposition 6*:

The profit function for the high risk market maker can be expressed as:

$$\Pi_H = (KO_H - B_H)A_H(O_H, O_L)$$

$$\frac{\partial \Pi_H}{\partial H} = (KO_H - B_H)\left\{\frac{\partial A_H}{\partial H} + \frac{\partial A_H}{\partial O_L}\cdot\frac{\partial O_L}{\partial H}\right\} - \frac{\partial B_H}{\partial H}A_H(O_H, O_L)$$

As an aside consider the following expressions:

$$A_H = 1 - \frac{O_H - O_L}{\alpha(H-L)}$$

$$\frac{\partial A_H}{\partial H} = -\left\{\frac{\alpha(O_H - O_L)}{[\alpha(H-L)]^2}\right\} = \frac{\alpha(O_H - O_L)}{[\alpha(H-L)]^2} = \frac{R}{H-L}$$

$$\frac{\partial A_H}{\partial O_L} = \left\{\frac{O_L'}{\alpha(H-L)}\right\} = \frac{1}{\alpha(H-L)}$$

$$\frac{\partial O_L}{\partial H} = \frac{\alpha K + B_H'}{3K}$$

Now substitute these expressions into the derivative of the profit function to get:

$$\frac{\partial \Pi_H}{\partial H} = (KO_H - B_H)\left\{\frac{R}{H-L} + \frac{\alpha K + B_H'}{3\alpha K(H-L)}\right\} - B_H'(1-R)$$

$$\frac{\partial \Pi_H}{\partial H} = \frac{(KO_H - B_H)[3\alpha KR + \alpha K + B_H'] - 3B_H'(1-R)\alpha K(H-L)}{3\alpha K(H-L)}$$

$$\frac{\partial \Pi_H}{\partial H} = \left\{\frac{(KO_H - B_H)(\alpha K + B_H') + 3B_H' R\alpha K(H-L)}{3\alpha K(H-L)}\right\}$$

$$+\left\{\frac{(KO_H - B_H)3\alpha KR - 3\alpha KB_H'(H-L)}{3\alpha K(H-L)}\right\}$$

Since the first term in {} is positive, we need to show that the second term of $\frac{\partial \Pi_H}{\partial H}$ is positive. That is:

$$(KO_H - B_H)3\alpha KR - 3\alpha KB_H'(H-L) \geq 0 \qquad (1)$$

$$(KO_H - B_H) = \left\{\frac{2\alpha K(H-L)}{3}\right\} + \left\{\frac{B_L - 2B_H}{3}\right\}$$

$$(KO_H - B_H) = \frac{2\alpha K(H-L) + B_L + 2B_H - B_H}{3}$$

$$\Longrightarrow \left\{ \frac{2\alpha K(H-L) + B_L - B_H}{3} \right\} R \geq B'_H(H-L)$$

$$R \geq \frac{3B'_H(H-L)}{2\alpha K(H-L) + B_L - B_H} \tag{2}$$

Since Assumption 2 implies that equation (2) is true, then it follows that equation (1) is true. This shows that the high risk market maker will want to deal in high risk assets as this increases its profitability.

**Low Risk Market Makers**:

$$\Pi_L = (KO_L - B_L)A_L(O_H, O_L)$$

$$\frac{\partial \Pi_L}{\partial L} = (KO_L - B_L)\left\{ \frac{\partial A_L}{\partial O_H} \cdot \frac{\partial O_H}{\partial L} + \frac{\partial A_L}{\partial L} \right\} - \frac{\partial B_L}{\partial L} A_L(O_H, O_L)$$

As an aside consider the following expressions:

$$A_L = \frac{O_H - O_L}{\alpha(H-L)}$$

$$\frac{\partial A_L}{\partial L} = -\frac{(O_H - O_L)(-\alpha)}{[\alpha(H-L)]^2} = \frac{\alpha(O_H - O_L)}{[\alpha(H-L)]^2} = \frac{R}{H-L}$$

$$\frac{\partial A_L}{\partial O_H} = \frac{1}{\alpha(H-L)}$$

$$\frac{\partial O_H}{\partial L} = -\frac{2\alpha K + B'_L}{3K}$$

Now substitute these expressions into the derivative of the profit function to get:

$$\frac{\partial \Pi_L}{\partial L} = (KO_L - B_L)\left\{ \frac{-2\alpha K + B'_L}{3\alpha K(H-L)} + \frac{R}{H-L} \right\} - B'_L R$$

$$\frac{\partial \Pi_L}{\partial L} = (KO_L - B_L)\left\{\frac{3\alpha KR - 2\alpha K + B'_L}{3\alpha K(H-L)}\right\} - B'_L R$$

If $\frac{\partial \Pi_L}{\partial L} < 0$ in addition to the result contained in equations (1) and (2) then a Nash equilibrium exists.

Now, we know that $-B'_L R < 0$. Thus, we now need to show that

$$(KO_L - B_L)\left\{\frac{3\alpha KR - 2\alpha K + B'_L}{3\alpha K(H-L)}\right\} < 0$$

Now note that since the offer price is greater than the bid price and since $K > 1$ the the term in $() > 0$, therefore we need to show:

$$3\alpha KR - 2\alpha K + B'_L < 0$$

$$\Leftrightarrow R < \frac{2\alpha K - B'_L}{3\alpha K}$$

which is consistent with Assumption (2), hence $\frac{\partial \Pi_L}{\partial L} < 0$

This proves that the low risk market maker will not try to increase his risk by trading high risk assets. Therefore, it will continue to trade low risk assets. Hence, the market makers will try to maximally differentiate themselves thus generating the Sub-game Perfect Nash Equilibrium.

In proving proposition 6, we showed that each market maker increases its profitability by positioning itself further away from the competition. However, the question of what happens to the profitability of the other market maker as it competitor varies its optimal choice remains to be answered, and is proven in Proposition 7.

*Proof of proposition 7*:

**High Risk Market Maker**

$$\frac{\partial \Pi_H}{\partial L} = (KO_L - B_L)\left\{\frac{\partial A_H}{\partial L} + \frac{\partial A_H}{\partial O_L}\frac{\partial O_L}{\partial L}\right\}$$

$$\frac{\partial A_H}{\partial O_L} = \frac{1}{\alpha(H-L)}, \qquad \frac{\partial O_L}{\partial L} = \frac{2B'_L - \alpha K}{3K}$$

$$\frac{\partial A_H}{\partial L} = \frac{(O_H - O_L)(-\alpha)}{\alpha^2(H-L)^2} = \left\{\frac{(O_H - O_L)}{\alpha(H-L)}\right\} \cdot \left\{\frac{-\alpha}{\alpha(H-L)}\right\} = \frac{-R}{H-L}$$

$$\frac{\partial \Pi_H}{\partial L} = (KO_H - B_H)\left\{\frac{2B'_L - \alpha K}{3\alpha K(H-L)} - \frac{R}{H-L}\right\}$$

$$= (KO_H - B_H)\left\{\frac{2B'_L - \alpha K - 3\alpha K R}{3\alpha K(H-L)}\right\}$$

Since $KO_H - B_H, \quad 3\alpha K(H-L) > 0$

We need to show $2B'_L - \alpha K - 3\alpha KR < 0$ (3)

But $3\alpha KR > 2B'_L - \alpha K$

$$\Rightarrow R > \frac{2B'_L - \alpha K}{3\alpha K}$$

which is validated by Assumption 2, hence equation (3) is true. The interpretation of the sign on the derivative is that If the low risk market maker tries to increase the risk on the assets it trades then this will have an adverse effect on the profitability of the high risk market makers. Hence, we can conclude that it is in the best interests of both market makers that the low risk market maker trades low risk assets.

**Low Risk Market Maker**

We now go on to show that if the high risk market maker increases its type then this will increase the low risk market maker's profitability. Recall, we already showed that the high risk type market maker increases its own profitability by doing this. So, we must ask ourselves, is $\frac{\partial \Pi_L}{\partial H} > 0$

$$\frac{\partial \Pi_L}{\partial H} = (KO_L - B_L)\left\{\frac{\partial A_L}{\partial H} + \frac{\partial A_L}{\partial O_H} \cdot \frac{\partial O_L H}{\partial H}\right\}$$

$$\frac{\partial A_L}{\partial O_H} = \frac{1}{\alpha(H-L)}, \qquad \frac{\partial O_H}{\partial H} = \frac{2\alpha K + 2B'_H}{3K}$$

$$\frac{\partial A_L}{\partial H} = -\frac{(O_H - O_L)\alpha}{(\alpha(H-L))^2} = -\frac{(R)}{(H-L)}$$

$$\frac{\partial \Pi_L}{\partial H} = (KO_L - B_L)\left\{\frac{-R}{H-L} + \frac{2\alpha K + 2B'_H}{3\alpha K(H-L)}\right\}$$

$$= (KO_L - B_L)\left\{\frac{-3\alpha KR + 2\alpha K + 2B'_H}{3\alpha K(H-L)}\right\}$$

We now show that $-3\alpha KR + 2\alpha K + 2B'_H > O$

But $3\alpha KR < 2\alpha K + 2B'_H$

$$\Leftrightarrow R < \frac{2\alpha K + 2B'_H}{3\alpha K}$$

which is in line with the Assumption 2 since

$$R < \frac{2\alpha K - B'_L}{3\alpha K} < \frac{2\alpha K + 2B'_H}{3\alpha K}$$

Thus, the low risk market maker's profitability will rise as the high risk market maker chooses to take on higher risks. Also, high risk market makers will experience increased profitability as low risk market makers get into assets constituted by lower levels of risks.

**Theorem 2**: *It is impossible for low risk market maker to mimic high risk market makers and be profitable.*

Proof of Theorem 2:

The lower boundary of Assumption 2 states the following:

$$0 < \frac{2B'_L - \alpha K}{3\alpha K} < R$$

$$\Leftrightarrow 2B'_L > \alpha K$$

$$\Leftrightarrow B'_L > \frac{\alpha K}{2}$$

From the upper boundary we have:

$$R < \frac{2\alpha K - B'_L}{3\alpha K} < \frac{2\alpha K + 2B'_H}{3\alpha K} < 1$$

$$\Leftrightarrow 2\alpha K + 2B'_H < 3\alpha K$$

$$\Leftrightarrow 2B'_H < \alpha K$$

$$\Leftrightarrow B'_H < \frac{\alpha K}{2}$$

$$\Rightarrow B'_H < \frac{\alpha K}{2} < B'_L$$

$$\Rightarrow B'_H < B'_L$$

Hence, it is more costly for the low risk market maker to try and trade in higher risk assets as this will reduce his profit.

# 4 Conclusion

Market makers specialize in different financial markets for different reasons. Specifically, it may not be feasible for a particular market maker to trade every type of financial instrument. This may be due to asymmetric information in the market place which can cause the market maker to price its assets incorrectly. This could result in arbitrage opportunities presenting themselves. Furthermore, the distribution of risk appetite of investors causes market makers to differentiate the products they offer to these investors.

Consequent on these realities, It has been shown that under a variable cost structure, the model that was examined results in maximal product differentiation as per D'Aspermont Gabszewick and Thissue, (1979). It is also shown that it would not be wise for the low risk market maker to try to imitate the high risk market maker by trying to trade high risk assets. That is, the low risk market maker is less efficient at trading high risk assets than the high risk market maker. As long as our assumptions hold the different types of market makers find it much more rewarding to maximally differentiate the products that they offer to investors. Competition is less fierce in a situation where there is maximal product differentiation. However, the compensation that can be received by maximally differentiating far outweighs that which can be achieved under more fierce competition.

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